TKR COLLEGE OF ENGINEERING AND TECHNOLOGY

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UNIT – I

QUANTUM MECHANICS AND BAND THEORY OF SOLIDS

Quantum Mechanics:

It is the branch of physics which explains about the motion of microscopic particles like electrons, protons etc. It was introduced by Max Planck in 1900.

Waves & particles:

Wave: A wave is nothing but spreading of disturbance in a medium. The Characteristics/ properties of waves are 1) Amplitude 2) Time period 3) Frequency 4) Wave-Length 5) Phase 6) Intensity.

Particle: A particle is a point in space which has mass & occupies space or region. The characteristics/properties of a particle are 1) Mass 2) velocity 3) Momentum 4) Energy etc. **Matter Waves or de-Broglie-waves:** The waves associated with a material particle are called as matter waves.

de-Broglie concept of dual nature of matter waves:

In 1924, Louis de-Broglie suggested that matter waves also exhibit dual nature like radiation (light). They are

I. Wave nature

II. Particle nature

Wave nature of matter waves is verified by Davisson & Germer experiment, G.P. Thomson experiment etc.

Particle nature of matter waves is verified by photo-electric effect, Compton effect etc.

de-Broglie hypothesis:

1) The universe consists of matter and radiation(light) only
2) Matter waves also exhibit dual nature like radiation.
3) The waves associated with the material particles are called as de-Broglie waves or matter waves & the wavelength associated with matter waves are called as de-broglie wavelength or matter wavelength (λ).
4) de-Broglie wave-length is given by $\lambda = \frac{h}{p} = \frac{hc}{\lambda}$

Expression for de-Broglie wavelength (λ) in various form:

According to the Planck's theory of radiation, the energy of photon is given by

$$E = h\nu = \frac{hc}{\lambda} \qquad (1)$$

h - Planck's constant, ν - frequency of photon

According to Einstein mass energy relation,

$$E = mc^2 \qquad (2)$$

m - mass of a photon, c -velocity of light

From equation of (1) & (2) $\frac{hc}{\lambda} = mc^2$

$$\lambda = \frac{h}{p} = \frac{h}{mv} \qquad (3)$$

Where p-momentum of photon = mv, m-mass of photon, c-speed of light

But according to de-Broglie theory Momentum of electron particle p = mv

M-mass of electron, v-velocity of electron, de-Broglie wavelength $\lambda = \frac{h}{p} = \frac{h}{mv}$

Eq. (4) gives the expression for de-Broglie wavelength.

Other forms of de-Broglie wavelength (λ):

(i) In terms of Energy(E):

We know that the kinetic energy of particle i.e. $E = \frac{1}{2}mv^2$

Multiply both sides with 'm'

$$2mE = m^2v^2$$

$$mv = \sqrt{2mE}$$

$$\frac{h}{\lambda} = \sqrt{2mE}$$

$$\lambda = \frac{h}{\sqrt{2mE}}$$

(ii) de-Broglie wavelength in terms of voltage (V) :

If a charged particle is accelerated through a potential difference (*V*), then the kinetic energy of the particle is given as

$$E = eV \tag{7}$$

But we have kinetic energy (*E*) of particle i.e. $E = \frac{1}{2}mv^2$

$$eV = \frac{1}{2}mv^2$$

Multiply by *'m'* on both sides we get

$$2meV = m^2v^2$$

$$mv = \sqrt{2meV}$$

$$\lambda = \frac{h}{\sqrt{2meV}}$$

Where $h = 6.625\times 10^{-34}$ Js, $m = 9.1 \times 10^{-31} Kg$, $e = 1.6 \times 10^{-19} c$

Therefore,

$$\lambda = \frac{h}{\sqrt{2meV}} \text{ or } \frac{12.27}{\sqrt{V}} \mathring{A}$$

Postulates of Quantum Mechanics:

According to classical mechanics, the state of a particle is specified, at any time t, by two fundamental dynamical variables: the position $r(t)$ and the momentum $p(t)$.

The quantum mechanical counterparts to these ideas are specified by postulates, which enable us to understand:

- how a quantum state is described mathematically at a given time t,
- how to calculate the various physical quantities from this quantum state, and
- knowing the system's state at a time t, how to find the state at any later time t^2 that is, how to describe the time evolution of a system.

1. **State of system**

The state of any physical system is specified, at each time t, by a state vector $|\psi(t)\rangle$ in a Hilbert space H; $|\psi(t)\rangle$ contains (and serves as the basis to extract) all the needed information about the system. Any superposition of state vectors is also a state vector.

2. Observables and operators

To every physically measurable quantity A, called an observable or dynamical variable, there corresponds a linear Hermitian operator $\hat{A}$ whose eigenvectors form a complete basis.

3. Measurements and eigenvalues of operators

The measurement of an observable A may be represented formally by the action of $\hat{A}$ on a state vector $|\psi(t)\rangle$.

4. Time evolution of a system

The time evolution of the state vector $|\psi(t)\rangle$ of a system is governed by the time- dependent Schrödinger equation

$$i\hbar\frac{\partial|\psi(t)\rangle}{\partial t} = H|\psi(t)\rangle$$

Operators in Quantum mechanics:

An operator is a mathematical rule that acts on a wave function ψ to produce another function. Every physical quantity (observable is a dynamical variable that can be measured) is represented by an operator. i.e position, momentum, angular momentum, energy.

The operator corresponding to the Hermitian is given by

$$\widehat{H} = \frac{P^2}{2m} + V(\vec{r}, t)$$

$$\widehat{H} = -\frac{\hbar^2\nabla^2}{2m} + V(\vec{r}, t)$$

Table 3.1 Some observables and their corresponding operators.

Observable	Corresponding operator
$\vec{r}$	$\hat{\vec{R}}$
$\vec{p}$	$\hat{\vec{P}} = -i\hbar\vec{\nabla}$
$T = \frac{p^2}{2m}$	$\hat{T} = -\frac{\hbar^2}{2m}\nabla^2$
$E = \frac{p^2}{2m} + V(\vec{r}, t)$	$\hat{H} = -\frac{\hbar^2}{2m}\nabla^2 + \hat{V}(\hat{\vec{R}}, t)$
$\vec{L} = \vec{r} \times \vec{p}$	$\hat{\vec{L}} = -i\hbar\hat{\vec{R}} \times \vec{\nabla}$

Expectation Values

The expectation value $\langle\widehat{A}\rangle$ of $\widehat{A}$ with respect to a state $|\psi\rangle$ is defined by

$$<\psi> = \frac{\langle\Psi|\hat{A}|\Psi\rangle}{\langle\Psi|\Psi\rangle}$$

The expectation value $\langle\hat{A}\rangle$ represents the average result of measuring A on the state $|\psi\rangle$ if the state is described by a wave function $\psi(x)$ then

$$\langle\hat{A}\rangle = \int_{-\infty}^{+\infty} \Psi^*(x)\widehat{A}\Psi(x)dx$$

Examples

Position $\langle\hat{x}\rangle = \int_{-\infty}^{+\infty} \Psi^*(x)\hat{x}\Psi(x)dx$

Momentum $\langle\hat{p}\rangle = \int_{-\infty}^{+\infty} \Psi^*(x)\left\langle -i\hbar\frac{\partial}{\partial x}\right\rangle\Psi(x)dx$

Energy $\langle\hat{E}\rangle = \int_{-\infty}^{+\infty} \Psi^*(x)\left\langle i\hbar\frac{\partial}{\partial t}\right\rangle\Psi(x)dx$

Eigen value & Eigen function:

A wave function Ψ, which satisfies all the properties is said to be *Eigen function* (Eigen =

proper)

An operator $\hat{O}$ is a mathematical operator (differentiation, integration, addition, multiplication, division etc.) which may be applied on a function Ψ(x), which changes the function to another function Φ(x). This can be represented as

$$\hat{O}\Psi(x) = \Phi(x)$$

If a function is Eigen function, then by result of operation with an operator $\hat{O}$, we get the same function as

$$\hat{O}\Psi(x) = \lambda\Psi(x)$$

$\Psi(x)$ – eigen function, λ – eigen value, $\hat{O}$- operator and $\Psi(x)$ =operand

Eg., $-\frac{d^2(sin2x)}{dx^2} = 4(sin2x)$, here $\hat{O} = -\frac{d^2}{dx^2}$; $\lambda = 4$; $\Psi(x) = sin2x$

Heisenberg uncertainty principle:

The discovery of dual nature of material particle imposes a serious hurdle in locating the exact position and momentum of a particle simultaneously.

This wax was removed by **"Werner. Heisenberg"** in 1927 by proposing a significant principle. Later it was called Heisenberg's uncertainty principle or Heisenberg's principle of indeterminacy.

"It is impossible to measure both the position and momentum of a particle simultaneously to any desired degree of accuracy." If 'Δx' & 'Δp' are uncertainties in the measurement of position & momentum of the particle then mathematically this uncertainties of this physical variables is written as

$$\Delta x.\Delta p \geq \frac{h}{4\pi} \qquad (1)$$

Similarly the uncertainties in measuring energy and time interval we can write

(i) $\Delta E.\Delta t \geq \frac{h}{4\pi}$ and the uncertainties in measuring angular momentum & angular displacement as

(ii) $\Delta J.\Delta\theta \geq \frac{h}{4\pi}$

Explanation:

(i) If $\Delta x = 0$. i.e., the position of a particle is measured accurately, then from eq-(1).

$$\Delta p = \frac{h}{\Delta x.4\pi} = \frac{h}{0} = \infty$$

It means that, the momentum of the particle can't be measured.

(ii) If $\Delta p = 0$. i.e., the momentum of a particle is measured accurately, from eq. (1).

$$\Delta x = \frac{h}{\Delta p.4\pi} = \frac{h}{0} = \infty$$

From the above said observations made by Heisenberg, he clearly states that it is impossible to design an experiment to prove the wave & particle nature of matter at any given instant of time.

If one measures position or momentum accurately, then there will be an uncertainty in the other. Thus, the Heisenberg's uncertainty principle gives the probability of determining the particle at any given instant of time in place of certainty.

Applications:

(i) It explains the non-existence of electron in the nucleus.

(ii) It gives the binding energy of an electron in atom.

(iii) It calculates the radius of Bohr's first orbit.

Physical significance of wave-function or Eigen − function ($\boldsymbol{\psi}$):

It is a variable or complex quantity that is associated with a moving particle at any position (x, y, z) and at any time't'.

(i) 'ψ' of a particle is represented by $\psi = \psi_0 e^{-i\omega t}$

(ii) 'ψ' explains the motion of microscopic particles.

(iii) 'ψ' is a complex quantity & it does not have any meaning.

(iv) $|\psi|^2 = \psi^*\psi$ is real and positive, it has physical meaning.

(v) $|\psi|^2$ represents the probability of finding the particle per unit volume.

(vi) For a given volume $d\tau$, the probability of finding the particle is given by probability density (ρ) = $\iiint |\psi|^2 d\tau$ where $d\tau$ = dxdydz

(vii) 'ψ' gives the information about the particle behavior.

(viii) ρ values are between 0 to 1.

(ix) Wavefunction 'ψ' is a single valued, finite and periodic function.

(x) If $\rho = \iiint |\psi|^2 d\tau = 1$, then '$\psi$' is called normalized wave function.

BAND THEORY OF SOLIDS

Free electron theory:

In solids, electrons in outer most orbits of atoms determine its electrical properties. Electron theory is applicable to all solids, both metals and non-metals. In addition, it explains the electrical, thermal and magnetic properties of solids. The structure and properties of solids are explained employing their electronic structure by the electron theory of solids.

It has been developed in three main stages:

1. Classical free electron theory
2. Quantum Free Electron Theory.
3. Zone Theory.

1. **Classical free electron theory:** The first theory was developed by Drude & Lorentz in 1900. According to this theory, metal contains free electrons which are responsible for the electrical conductivity and metals obey the laws of classical mechanics.

Assumptions:

- Electrons behave like classical particles
- Electron–electron interaction is neglected
- Collisions occur between electrons and ions

Success:

- Explains Ohm's law
- Gives expression for electrical conductivity
- Explains thermal conductivity qualitatively

Failures:

- Cannot explain temperature dependence correctly
- Fails to explain specific heat of metals
- Cannot explain photoelectric effect and other quantum phenomena

2. **Quantum Free Electron Theory**: In 1928 Sommerfield developed the quantum free electron theory. According to Sommerfield, the free electrons move with a constant potential. This theory obeys quantum laws.

Assumptions:

- Electrons move freely inside a metal box
- Energy levels are quantized
- Pauli exclusion principle is obeyed

Success:

- Correctly explains electrical conductivity
- Explains electronic specific heat
- Introduces Fermi energy and Fermi level
- Explains temperature dependence more accurately

Failures:

- Does not consider periodic potential of lattice.
- Cannot explain band gaps in insulators and semiconductors

3. **Zone Theory**: Bloch introduced the band theory in 1928. According to this theory, free electrons move in a periodic potential provided by the lattice. This theory is also called "Band Theory of Solids". It gives complete informational study of electrons.

Bloch Theorem:

According to free electron model, a conduction electron in metal experiences constant potential. But in real crystal, there exists a periodic arrangement of positively charged ions through which the electrons move. As a consequence, the potential experienced by electrons is not constant but it varies with the periodicity of the lattice. In zone theory, as per Bloch, potential energy of electrons considered as varying potential with respect to lattice 'a'.

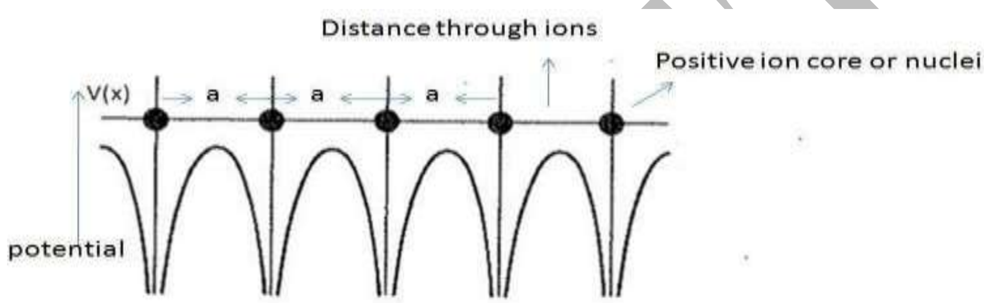


Fig. Variation of potential energy in a periodic lattice.

Let us examine one dimensional lattice as shown in figure. It consists of array of ionic cores along X-axis. A plot of potential V as a function of its position is shown in figure.

From graph:

At nuclei or positive ion cores, the potential energy of electron is minimum and in-between nuclei, the P.E.is considered as maximum w.r.to. Lattice constant 'a'.

This periodic potential V(x) changes with the help of lattice constant a,V(x) = $V\ (x + a)$

('a' is the periodicity of the lattice)

To solve, by considering Schrodinger's time independent wave equation in one dimension,

$$\frac{d^2y}{dx^2} + \frac{8\pi^2 m}{h^2}[E - V(x)]\Psi = 0 \qquad (1)$$

Bloch's 1D solution for Schrodinger wave equation $\psi\,(x) = \,(x)exp(ikx)$ (2)

where $u_k(x) = u_k(x + a)$

Here $u_k(x)$ periodicity of crystal lattice, modulating function, k- propagation vector $\frac{2\pi}{\lambda}e^{ikx}$ is plane wave.

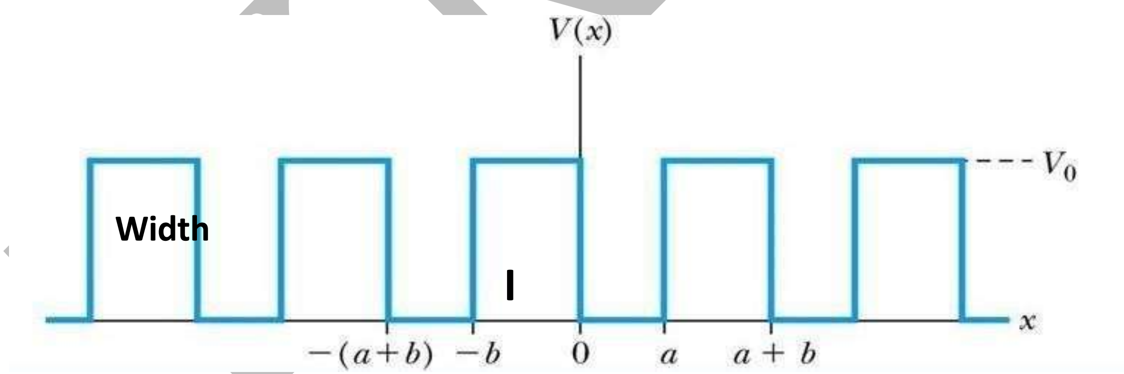
By applying eq^n(2) to eq^n(1), it is not easy to solve Schrodinger wave equation and Bloch cannot explains complete physical information about an electron in periodic potential field. Then Kronig-Penny model was adopted to explains the electrical properties of an electron.

Kronig-Penney model:

Kronig - penney approximated the potentials of an electrons inside the crystal in terms of the shapes of rectangular steps as shown, i.e. square wells is known as Kronig Penny model.i.e. The periodic potential is taken in the form of rectangular one-dimensional array of square well potentials and it is the best suited to solve Schrodinger wave equation.

It is assumed that the potential energy is zero when x lies between 0 and a, and is considered as I region. Potential energy is V_0, when x lies between –b and 0. And considered as II region.

Boundary conditions:



$$V(x) = 0\,, \textit{where x lies between } 0 < x < a \; - \; I\ \textit{region}$$

$$V(x) = V_0, \textit{where x lies between } -b < x < 0 \; - \; II\ \textit{region}$$

This model explains many of the characteristic features of the behavior of electrons in a periodic lattice.

The wave function related to this model may be obtained by solving Schrodinger equations for the two regions,

$$\frac{d^2\Psi}{dx^2} + \frac{2m}{\hbar^2} E\Psi = 0, \text{ for } 0 < x < a \text{ with } V(x)=0 \quad (1)$$

$$\frac{d^2\Psi}{dx^2} + \frac{2m}{\hbar^2}[E - V_0]\Psi = 0, \text{ for } -b < x < 0 \text{ with } V(x)=V_0 \quad (2)$$

Again,

$$\frac{d^2\Psi}{dx^2} + \alpha^2\Psi = 0 \quad (3)$$

$$\frac{d^2\Psi}{dx^2} - \beta^2\Psi = 0 \quad (4)$$

Where $\alpha^2 = \frac{2mE}{\hbar^2}$ and $\beta^2 = \frac{2m}{\hbar^2}[V_0 - E]$

The solution of these equations from Bloch theorem, $u_k(x) = u_k(x + a)$. From figure, square well potentials, if V_0 increases, the width of barrier 'w' decreases, if V_0 decreases the width of barrier w increases. But the (product) barrier strength $V_0 w$ remains constant.

To get this, differentiating above Schrodinger wave equations 3 & 4 w.r.to x, and by applying boundary conditions of x (w.r.to their corresponding **Ψ**), to known the values of constants

A, B of region -I, C, D-for region-II, we get mathematical expression (by simplification)

$$\cos ka = P\frac{sin\alpha a}{\alpha a} + cos\alpha a$$

Where $P = \frac{mV_0 ab}{\hbar^2}$ and $\alpha^2 = \frac{2\pi}{h}\sqrt{2mE}$

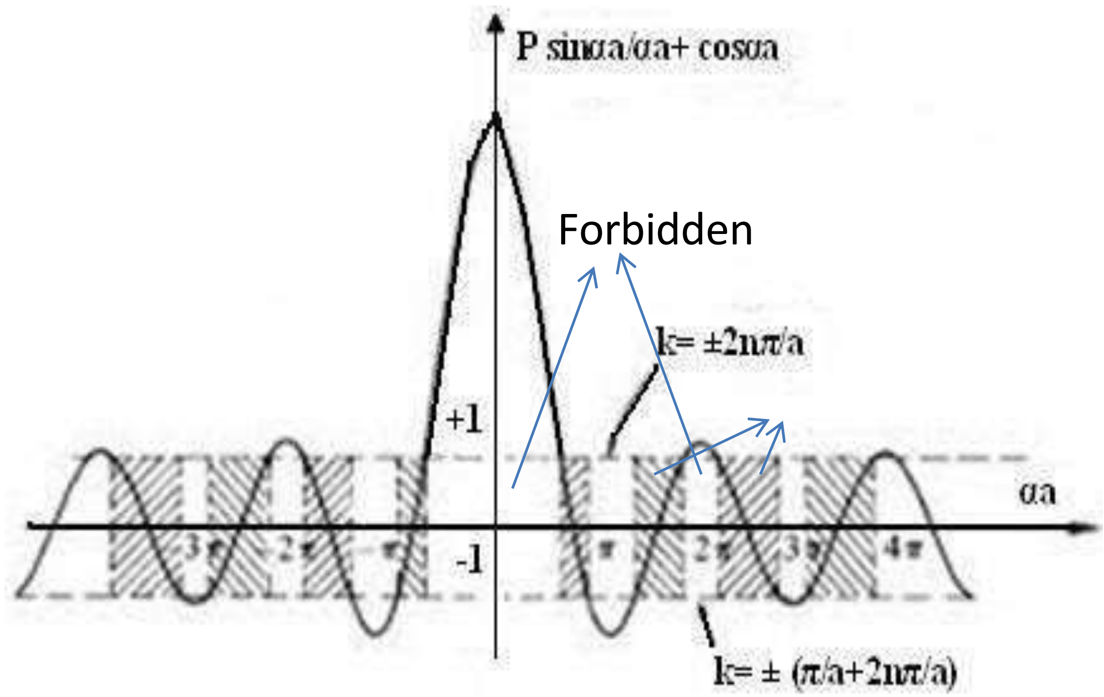
P-varying term, known as scattering power. and '$V_0 b$' is known as barrier strength.

Conclusions:

1. The L.H.S is a cosine term which varies between the limits -1 and +1, and hence the R.H.S also varies between these limits. It means energy is restricted within -1 to +1 only.
2. If the energy of electron lies between -1 to +1, are called **allowed energy bands** and it is shown by shaded portion in energy spectrum. This means that 'αa' can take only certain range of values belonging to allowed energy band.
3. As the value of αa increases, the width of the allowed energy bands also increases.
4. If energy ofe^{-}s not lies between -1 to +1 are known as **forbidden energy bands** and it decreases w.r.to increment of αa.

5. Thus, motion of electrons. in a periodic lattice is characterized by the bands of allowed & forbidden energy levels.

Case 1:



If $P \rightarrow \infty$, the allowed band reduces to a single (line) energy level, gives us steeper lines. We have $\cos ka = P\frac{\sin\alpha a}{\alpha a} + \cos\alpha a$

2 lines are not able to understand

$$\sin\alpha a = \sin n\pi$$

$$\alpha a = n\pi$$

$$\alpha^2 a^2 = n^2\pi^2$$

$$\alpha^2 = \frac{n^2\pi^2}{a^2}$$

$$\frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{a^2}$$

$$E = \frac{n^2\pi^2\hbar^2}{2ma^2} = \frac{n^2h^2}{8ma^2}$$

$her \quad is\ lattice\ constant$

It means, it (zone theory) supports quantum free electron theory.

Case 2:

$P \rightarrow 0$, We have

$$\cos ka = P\frac{sin\alpha a}{\alpha a} + cos\alpha a$$

$$\alpha a = ka$$

$$\alpha = k$$

$$\alpha^2 = k^2$$

$$\frac{2mE}{\hbar^2} = \left(\frac{2\pi}{\lambda}\right)^2 = \frac{4\pi^2}{\lambda^2}$$

$$E = \frac{4\pi^2\hbar^2}{2m\lambda^2}$$

$$E = \frac{1}{2}mv^2$$

It gives us kinetic energy of an electron. It means zone theory supports classical free electron theory at this situation electron completely free electron not bounded with allowed and forbidden gaps (and no energy level exists).

Thus, by varying P from 0 to ∞, we find that the completely free electron(s) becomes completely bound to Brillouin Zone**.**

E-K diagram:

The Brillouin zone are the boundaries that are marked by the values of wave vector k, in which electrons can have allowed energy values. These represent the allowed values of k of the electrons in 1D, 2D &3D.

We have, the energy of the electron in a constant potential box is,

$$E = \frac{n^2h^2}{8ma^2} \tag{1}$$

Where a = length of the box.

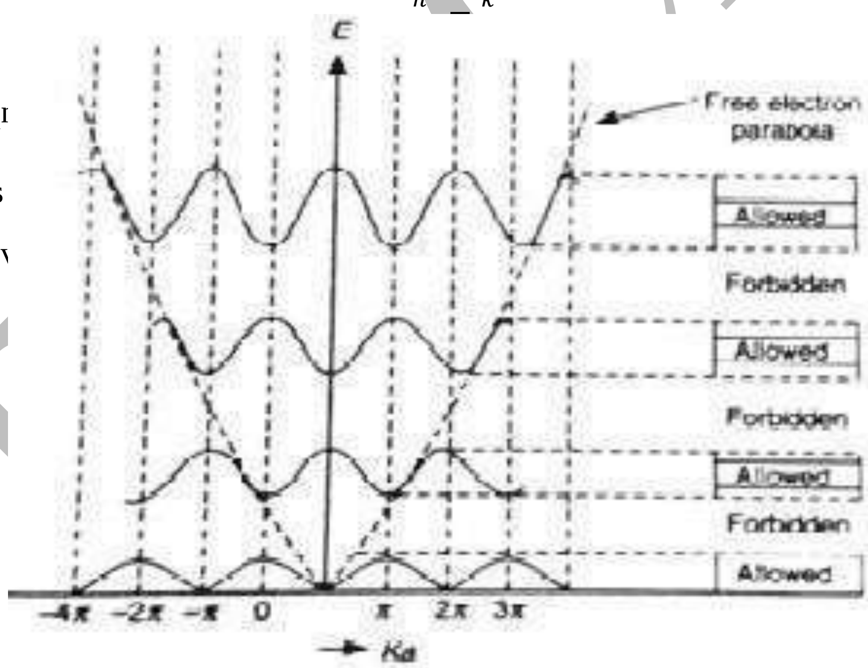
But, $= \frac{n\pi}{a}$; $k^2 = \frac{n^2\pi^2}{a^2}$

n^2 k^2 (2)

Substitute eq

It represents E) and the wave

vector k, for v



It is the energy spectrum of an electron moving in presence of a periodic potential field and is divided into allowed energy regions (allowed zones) or forbidden energy gaps (forbidden zones).

Allowed energy values lie in the region k=-π/a to =+π/a. This zone is called the first Brillouin zone. After a break in the energy values, called forbidden energy band, we have another allowed zone spread from k=-π/a to -2π/a and +π/a to +2π/a. This zone is called the second Brillouin zone. Similarly, higher Brillouin zones are formed.

Concept of effective mass of electron:

When an electron in a periodic potential of lattice is accelerated by an known electric field or magnetic field, then the mass of the electron is called effective mass and is represented by m^*

To explain, let us consider an electron of charge 'e' and mass 'm' moving inside a crystal lattice of electric field E.

Then by taking known expression F=ma, can be considered here as $F = m * a$ (1)

The acceleration $a = \frac{eE}{m}$ is not constant in the periodic lattice but varies due to the change in electronic mass.

If free electron under wave packet, the group velocity V_g corresponding to the particle's velocity can be written as

$$v_g = \frac{d\omega}{dk} = 2\pi\frac{d\nu}{dk} = \frac{2\pi}{h}\frac{dE}{dk} = \frac{1}{\hbar}\frac{dE}{dk} \quad (2)$$

Where $E = h\nu$

The rate of change of velocity is known as acceleration, $a = \frac{dv_g}{dt} = \frac{1}{\hbar}\frac{d^2E}{dkdt}$

$$a = \frac{1}{\hbar}\frac{dE}{dt}\frac{dk}{dk} = \frac{1}{\hbar}\frac{d^2Edk}{dk^2dt} \quad (3)$$

From quantum mechanics relation, $p = \hbar k$ (4)

And $F = \frac{dP}{dt}$ (5)

By differentiating eq.4 w.r.to 't' and by substituting eq.5

$$\hbar \frac{dk}{dt} = \frac{dp}{dt} = F \Rightarrow \frac{dk}{dt} = \frac{F}{\hbar} \quad (6)$$

by substituting eq.6 in eq.3,

$$a = \frac{1}{\hbar}\frac{d^2E}{dk^2}\frac{F}{\hbar}$$

By rearranging the above term and by comparing with eq.1

$$F = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$$

Is known as expression for m^* and it is depends on E and K

Origin of energy band formation in Solids:

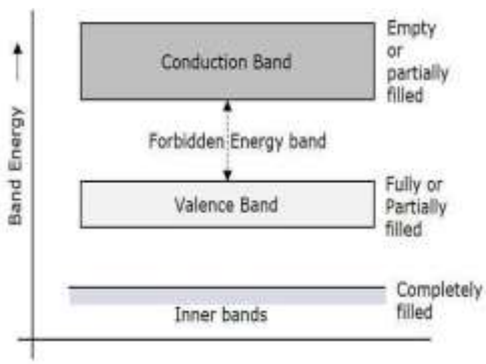
The band theory of solids explains the formation of energy bands and determines whether a solid is a conductor, semiconductor or insulator.

When two identical atoms are brought closer, the outermost orbits of these atoms overlap and interact. When the wave functions of the electrons of different atoms begin to overlap considerably, the energy levels corresponding to those wave functions split.

If more atoms are brought together more levels are formed and for a solid of N atoms, each of the energy levels of an atom splits into N energy levels. These energy levels are so close that they form an almost continuous band. The width of the band depends upon the degree of overlap of electrons of adjacent atoms and is largest for the outermost atomic electrons.

As a result of the finite width of the energy bands, gaps are essentially leftover between the bands called forbidden energy gap. The electrons first occupy the lower energy levels (and are of no importance) then the are of important to explain electrical properties of solids and these are called valence band and conduction band.

Valence band: A band occupied by valence electrons and is responsible for electrical, thermal and optical properties of solids and it is filled at 0K.



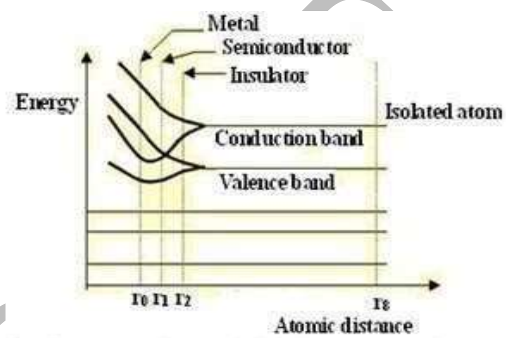


Fig: Formation of energy bands when atoms are closer

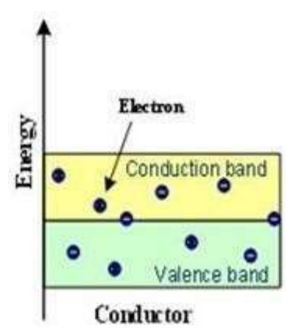
Conduction band: A band corresponding to outer most orbit is called conduction band and is the highest energy band and it is completely empty at 0K.

The forbidden energy gap between valence band conduction band is known as the energy band gap.

By this solids are classified in to conductors, semiconductors and insulators.

Classification of solids into conductors, Semiconductors & Insulators:

Based on the energy band diagram materials or solids are classified as follows:

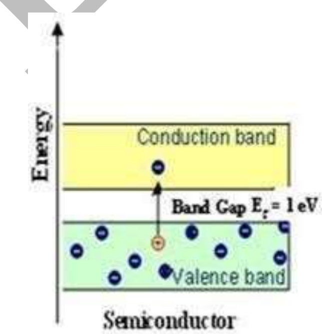


Conductors: In this kind of materials, there is no forbidden gap between the valence band and conduction band. It is observed that the valence band overlaps with the conduction band in metals as shown in figure. There are sufficient numbers of free electrons, available for electrical conduction and due to the overlapping of the two bands there is an easy transition of electrons from one band to another band takes place, and there no chance for the presence of holes. Resistivity of conductors is very small and it is very few milli ohm meters.(Ωm).

Examples: All metals (Na, Mg, Al, Cu, Ni Cu, Ag, Li, Ar etc)

Semiconductors:

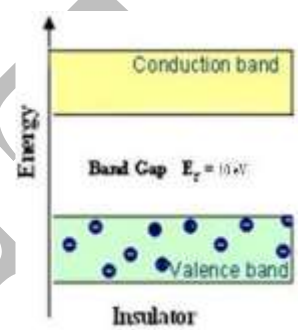
In semiconductors, there is a band gap exists between the valence band and conduction band and it is very less and it is the order of - 1 to 2 eV are known as semiconductors.It will conduct electricity partially at normal conditions. The electrical resistivity values are 0.5 to 10^3 ohm meter. Due to thermal vibrations within the solid, some electrons gain enough energy to overcome the band gap (or barrier) and behave as conduction electrons. Conductivity exists here due to electronics and holes.



Examples: Silicon, Germanium, Ga As.

Insulators:

In insulators, the width of forbidden energy gap between the valence band and conduction band is very large. Due to large energy gap, electrons cannot jump from V.B to C.B. Energy gap is of the order of ~ 10 eV and higher than semiconductors. Resistivity values of insulators are 10^7 to 10^{12} ohm-m. Electrons are tightly bound to the nucleus, no valence electrons are available. Examples: Wood, rubber, glass.



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UNIT – II

Crystallography and Nano Materials & Characterization

Introduction to Crystallography:

Crystallography is the experimental science of the arrangement of atoms in solids. The word "crystallography" derives from the Greek words crystallon = cold drop / frozen drop, with its meaning extending to all solids with some degree of transparency, and grapho = write.

Solid state physics is a branch of physics which deals with the study of structure, properties and applications of solids.

States of matter and types of solids

Matter basically exists in three states- solid, liquids, gases. The other two states are plasma and liquid crystal.

Solid State is a solid consist of atoms or cluster of atoms arranged in close proximity. The physical structure of a solid and its properties are closely related to the scheme of arrangement of atoms within the solid.

Solids can be classified into two categories namely crystalline and amorphous solids**.**

Crystalline materials

In crystalline solids the atoms are arranged in a periodic manner and the atomic arrangement is found to be uniform throughout the material. There are single and polycrystalline materials. In polycrystalline materials, each of the constituent crystal has uniform atomic arrangement with in its boundary.

Examples: Quartz, Calcite, Sugar, Mica, Diamonds, table salt etc.

Non-crystalline materials or Amorphous solids

In amorphous solids, the atoms are arranged in a random manner and there is no periodic atomic arrangement in any part of the material.

Examples: Plastics, Glass, Rubber, Metallic Glass, Polymers, Gel etc.

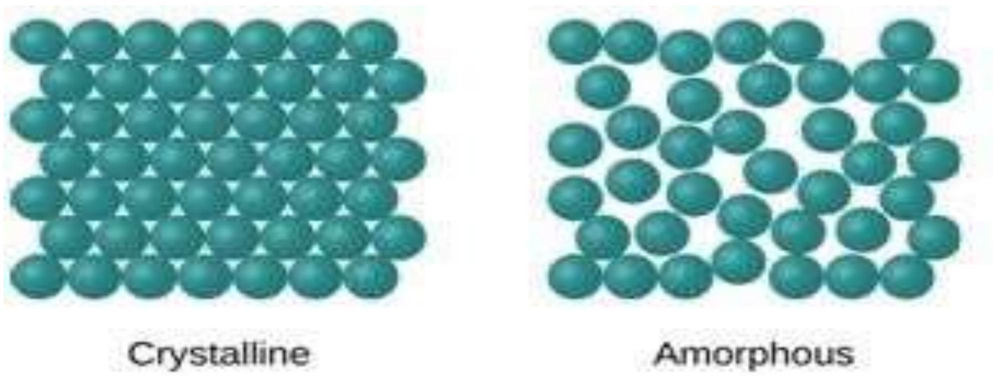
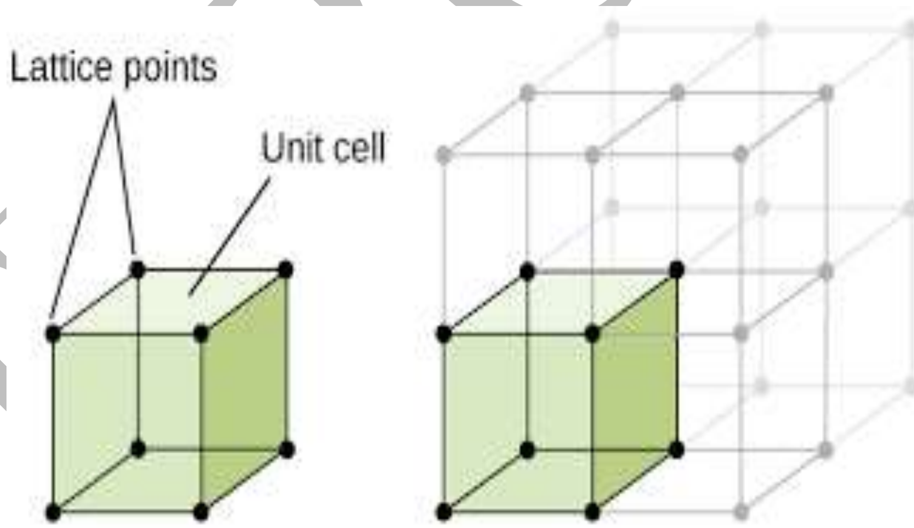


Figure 1.1: crystalline and amorphous solids

Unit cell

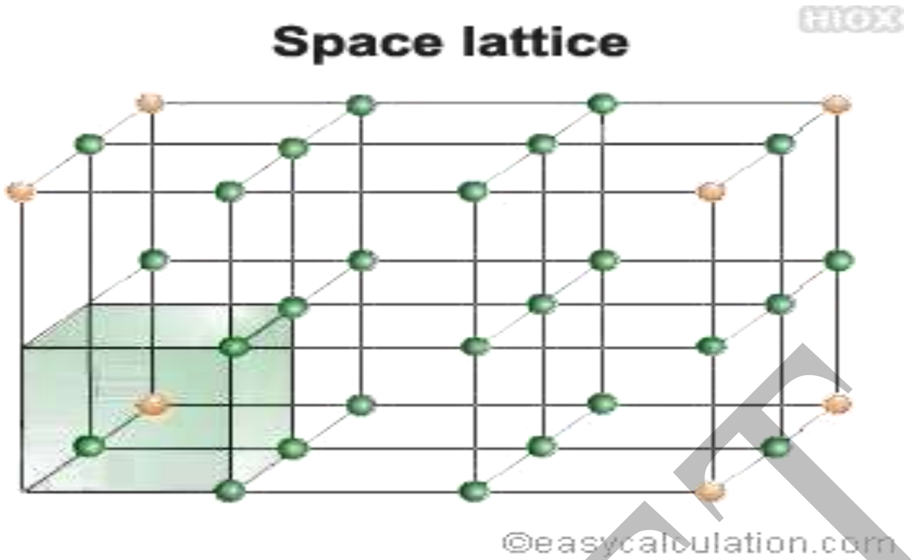
The smallest fundamental building block of the space lattice is called a unit cell. It has the minimum volume. The regular periodic repetition of this unit cell in a three-dimensional space generates the complete space lattice.



Space lattice

A space lattice is defined as array of points arranged in a regular periodic fashion in three-dimensional space such that each point in the lattice has identical surroundings.

The points are called lattice points or lattice sites**.**



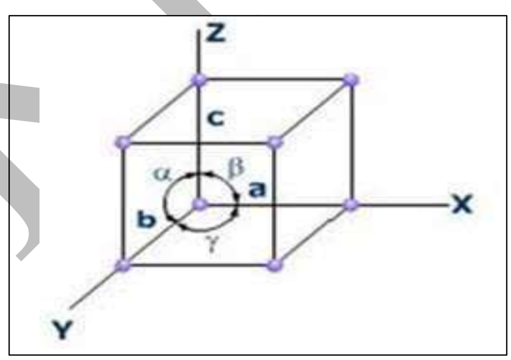
Basis

An atom or a group of atoms that is associated with every lattice point is identical in composition, arrangement and orientations is called basis

lattice parameters

The axial lengths a, b, c and the three inter-axial or interfacial angles α, β, γ are the basic lattice parameters.

Unit cell is characterized by six lattice parameters i.e. three lattice constants along three crystallographic axes (cube edges) a, b, c and three interfacial angles α, β, γ.



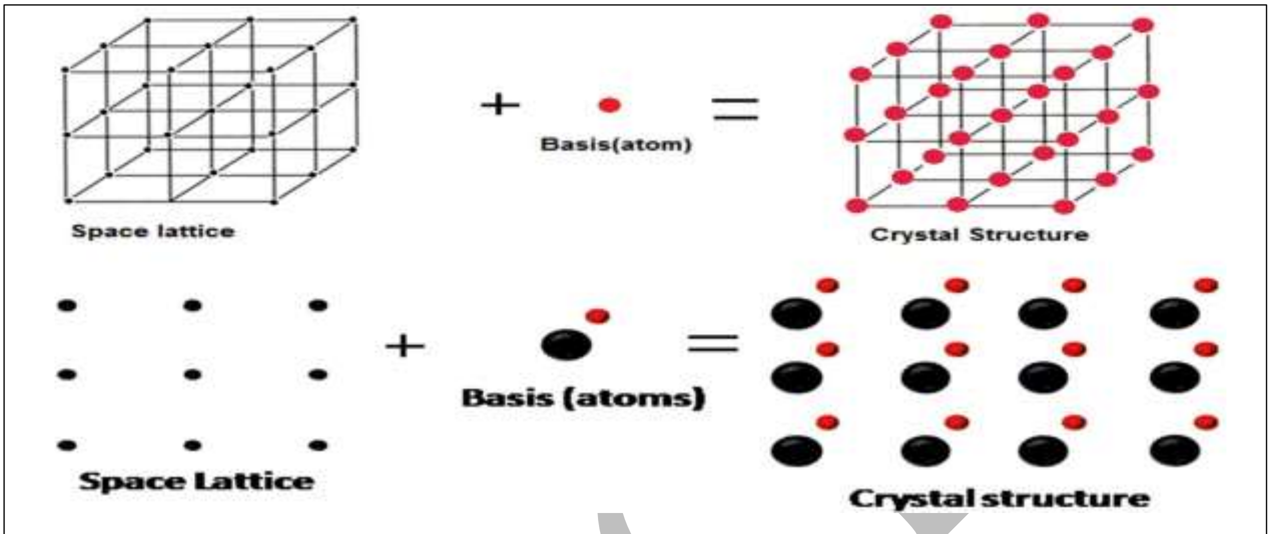
Lattice parameters of the unit cell

Crystal structures

Crystal lattice or crystal structure is defined as a space lattice in which lattice sites are occupied by atoms or clusters of atoms (basis) so each lattice point is associated with the same unit of group of atoms.

Crystal structure is obtained when a basis is added at each point in the lattice. Thus crystal structure is therefore the result of two quantities; namely space lattice and a basis. Figure shows the basis consisting of a group of two atoms.

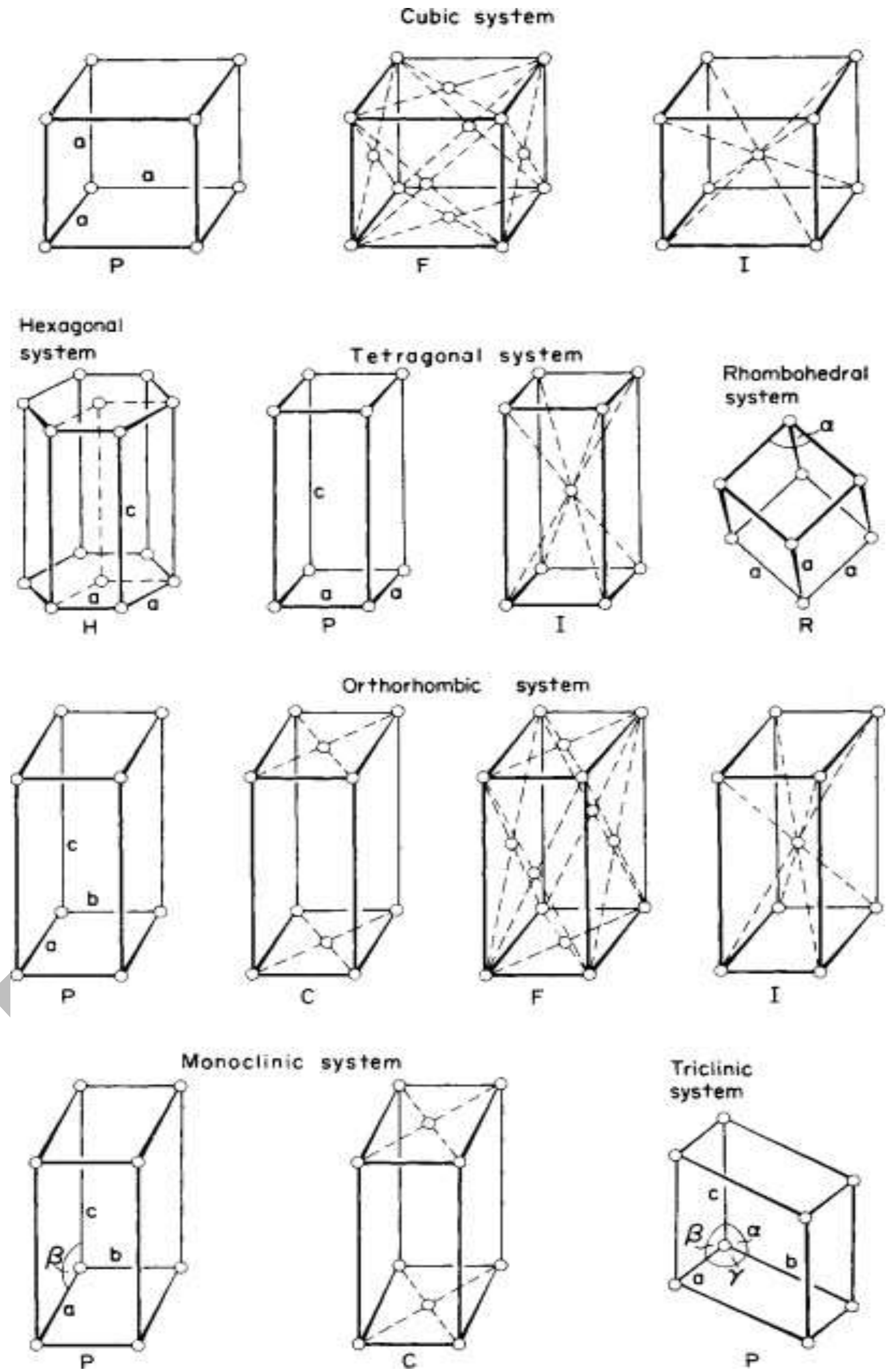
Space Lattice + Basis = Crystal Structure



Bravais lattices

French Crystallographer **Bravais**, proposed that there are only 14 possible ways in which lattice points can be arranged in a regular periodic manner in three-dimensional space. These 14 different arrangements are called Bravais Lattices. These Bravais lattices are grouped into seven crystal systems namely cubic, monoclinic, triclinic, orthorhombic, trigonal (rhombohedral), tetragonal and hexagonal.

Crystal System	Lattice Parameters	No. of Bravais Lattices	Types of Lattices
1. Cubic	$a = b = c$; $\alpha = \beta = \gamma = 90°$	3	(i) Simple Cubic (SC) (ii) Body-Centered Cubic (BCC) (iii) Face-Centered Cubic (FCC)
2. Tetragonal	$a = b \neq c$; $\alpha = \beta = \gamma = 90°$	2	(i) Simple Tetragonal (ii) Body-Centered Tetragonal
3. Orthorhombic	$a \neq b \neq c$; $\alpha = \beta = \gamma = 90°$	4	(i) Simple Orthorhombic (ii) Body-Centered Orthorhombic (iii) Face-Centered Orthorhombic (iv) Base-Centered Orthorhombic
4. Hexagonal	$a = b \neq c$; $\alpha = \beta = 90°$, $\gamma = 120°$	1	(i) Simple Hexagonal
5. Rhombohedral (Trigonal)	$a = b = c$; $\alpha = \beta = \gamma \neq 90°$	1	(i) Simple Rhombohedral
6. Monoclinic	$a \neq b \neq c$; $\alpha = \gamma = 90°$, $\beta \neq 90°$	2	(i) Simple Monoclinic (ii) Base-Centered Monoclinic
7. Triclinic	$a \neq b \neq c$; $\alpha \neq \beta \neq \gamma \neq 90°$	1	(i) Simple Triclinic


Cubic system
P
F
I
Hexagonal system
H
Tetragonal system
P
I
Rhombohedral system
R
Orthorhombic system
P
C
F
I
Monoclinic system
P
C
Triclinic system
P

For cubic system, three types of unit cells are as shown in figure 5.

1) **Simple cubic (sc):**

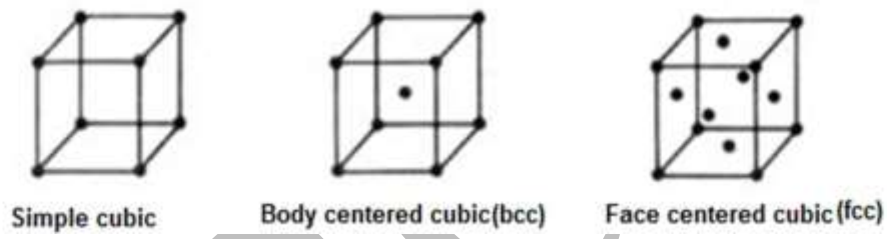
A simple Cubic Cell is a primitive Cell with lattice points located at eight corners of the cube.

2) **Body centered cubic (bcc):-**

The Body Centered Cubic Cell is a non-primitive cell having a lattice point within the cell in addition to the eight lattice points at eight corners of the cube.

3) **Face centered cubic (fcc):-**

The Face Centered Cubic Cell is a non-primitive cell having six lattice points at the centers of its six faces and eight lattice points at eight corners of the cube.



THE UNIT CELL CHARACTERISTICS

Unit cell Volume, V

The volume of a unit cell is given by

$V = abc$

when a=b=c, as in the case of cubic cells

$V = a^3$

Effective number of atoms per unit cell (Z) / Rank of Unit cell:-

The effective number of atoms per unit cell is equal to the product of the number of atoms per lattice point and the number of lattice points per unit cell. Unit cell is a part of an infinite scheme and is not an isolated entity. Therefore, the Lattice points do not exclusively belong to one unit cell but each one is shared by several adjacent cells. As a result an atom located at a lattice site contributes only a fraction of its mass and volume to one unit cell.

Atomic Radius (r):-

The relationship between the apparent size of the atom and the edge of the unit cell can be determined where one atom is attached to a lattice site.

Nearest neighbour distance (2r):

The nearest neighbor distance, in metallic structures is the distance between the centers of the atoms in contact expressed in terms of the length of the edge of the unit cell

Coordination number (CN):

The coordination number of an atom in a crystal is the number of nearest neighbour atoms which are simultaneously in contact with that atom. It signifies the tightness of packing of atoms in the crystal.

Atomic packing fraction (APF):

The fraction of the space occupied by atoms in the unit cell is known as atomic packing fraction. **It is defined as the ratio of the volume of effective number of atoms in the unit cell to the total volume of the unit cell. Thus,**

$$APF = \frac{\left({}^{No.of\ atoms}\!/_{unitcell} \right) (volume\ of\ each\ atom)}{Volume\ of\ the\ unit\ cell}$$

The atomic packing fraction denotes the efficiency with which the available space in a unit cell is utilized.

Void space:

The void space in the unit cell is the vacant space left unutilized in the cell. It is equal to (1-APF). It is often expressed as %. It is commonly known as <u>interstitial space</u>.

Thus, **%Void space= (1 – APF) x 100**

Density: - The density of unit cell must be the same as that of bulk crystal.

$$Density\ \rho = \frac{ZW}{V}$$

Where W is the mass of each atom

$$W = \frac{M}{N}$$

M → the molecular weight of the material & N→ the Avogadro number.

Z is effective number of atoms per unit cell.

V is volume of unit cell.

$$Density\ \rho = \frac{ZW}{V}$$

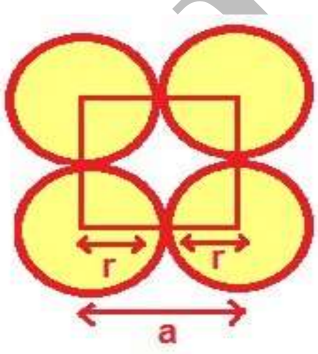
Simple Cubic Cell (SC) :

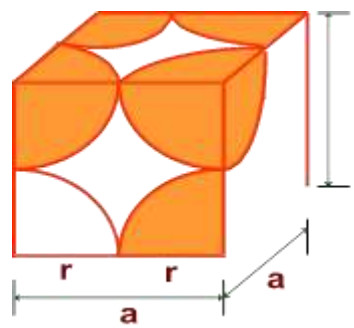
Unit cell Volume:-

The volume of a unit cell is given by If $\alpha = \beta = \gamma = 90^{o}$, then V = abc Further, when a=b=c, as in the case of cubic cells

$$V = a^3$$

Atomic radius:-





In a SC cell the atoms would be in contact along the edges of the cube, as shown in fig. If ‘a’ is the edge of the cubic cell & r be the radius of the atom.

$$\boldsymbol{a = 2r \quad or \quad r = \frac{a}{2}}$$

Effective number of atoms per unit cell (Z) :

A simple cubic cell has eight lattice point at its eight corners, which are occupied by eight atoms. In the three-dimensional array, each corner atom is linked to eight surrounding cells, hence in effect, each corner atom contributes $\left(\frac{1}{8}\right)^{th}$ of its content to a unit cell. hence one unit cell contains $\frac{1}{8} \times 8 = 1$ atom.

Coordination Number (CN):

Around an atom in a SC cell, there would be six equally spaced nearest neighbour atoms each at a distance 'a' from that atom. Four atoms lie in the plane of the atom while one is vertically above it and one vertically below. Therefore, coordination number, CN=6

Atomic Packing Fraction (APF):

$$APF = \frac{Zv}{V}$$

For SC cell, Z = 1

$v = \frac{4}{3}\pi r^3$ is the Volume of spherical atom

V = Volume of the unit cell= $a^3 = (2r)^3 = 8r^3$

$$APF = \frac{\left[1 \times \left(\frac{4}{3}\pi r^3\right)\right]}{8r^3} = \frac{\pi}{6} = 0.52$$

Percentage void space:

$$\%\,Void\ space = (1 - APF) \times 100$$

$$\%\,Void\ space = (1 - 0.52) \times 100 = 48\%$$

Examples:- polonium (Po)

<u>Body Centered Cubic Cell (BCC) :</u>

Unit cell Volume:

The volume of a unit cell is given by

V= abc

when a = b = c, as in the case of cubic cells V = a^3

Atomic Radius : In a BCC cell atom are in contact along the body diagonal. From

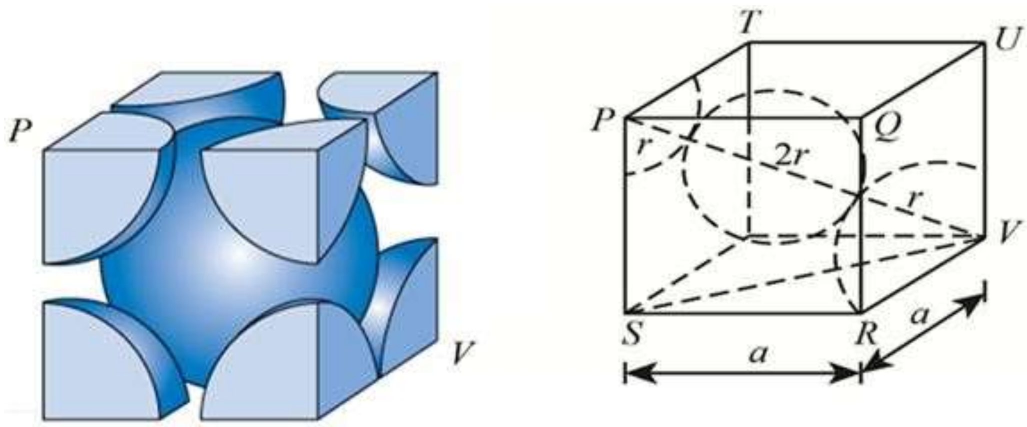


Figure 1.13: body centered cubic crystal.

$$PV^2 = SV^2 + PS^2 = 2a^2 + a^2 = 3a^2$$

$$4r^2 = 3a^2 \; or \; a = \frac{4}{\sqrt{3}}r$$

$$SV^2 = SR^2 + RV^2 = a^2 + a^2 = 2a^2$$

$$r = \frac{\sqrt{3}}{4}a$$

Effective Number of atom per unit cell (Z):

A BCC cell has eight atoms at corners of the cube and one atom within the volume of the cell. An atom at the lattice point within the cell belongs completely to the cell and the atoms at the corners of the cell contributes $\left(\frac{1}{8}\right)^{th}$ share. Therefore, the number of atoms in a BCC unit cell is $1 + 8 \times \left(\frac{1}{8}\right) = 2$

Coordination Number (CN):

In the BCC cell each corner atom is in contact with the body centre atom. As there are eight unit cells around each corner, the atom located at each corner would be simultaneously touching the eight-body centre atom around it. Therefore, coordination number is eight.

CN = 8

Atomic Packing Fraction:

$$APF = \frac{Zv}{V}$$

For BCC cell, Z = 2

$v = \frac{4}{3}\pi r^3$ is the Volume of spherical atom

V = volume of the unit cell

$$a^3 = \left(\frac{4r}{\sqrt{3}}\right)^3 = \frac{64r^3}{3\sqrt{3}}$$

$$APF = \frac{\left[2 \times \left(\frac{4}{3}\pi r^3\right)\right]}{\frac{64r^3}{3\sqrt{3}}} = \frac{\pi\sqrt{3}}{8} = 0.68$$

Percentage void space:

$$\%\,Void\ space = (1 - APF) \times 100$$

$$\%\,Void\ space = (1 - 0.68) \times 100 = 32\%$$

Density:

$$\rho = \frac{ZM}{N_A a^3}$$

In case of BCC cell, Z= 2

$$\rho = \frac{2M}{N_A a^3}$$

FACE CENTRED CUBIC CELL (FCC)

Unit cell Volume:

The volume of a unit cell is given by V= abc when a=b=c, as in the case of cubic cells V= a^3

Atomic Radius:

In a FCC cell, atoms are in contact along the face diagonal of the cube, as shown in fig

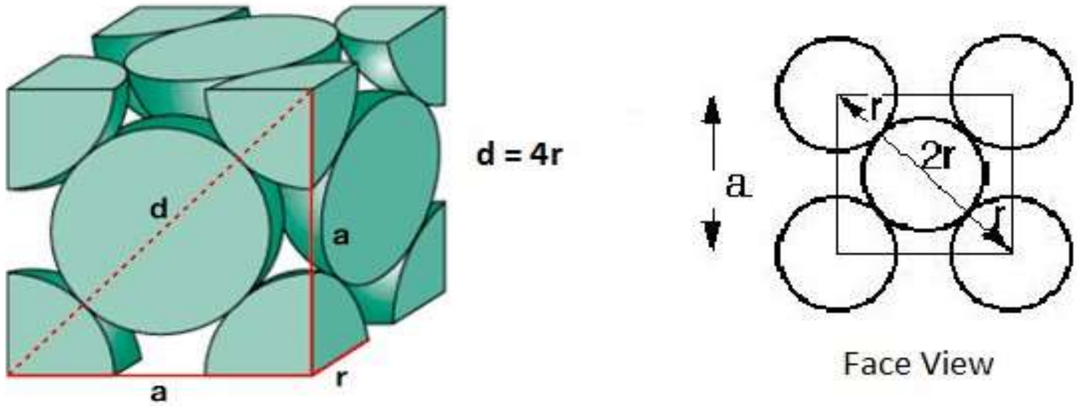


Figure 1.14.: Face centered cubic

$$4r^2 = a^2 + a^2 = 2a^2$$

$$r = \frac{\sqrt{2}}{4}a \quad or \quad r = \frac{a}{2\sqrt{2}}$$

Effective Number of atom per unit cell (Z): -

In a FCC cell, eight atoms are located at the eight corners and six atoms are at the centres of the six faces. Therefore, each face centered atom contributes only half of its mass & volume to one cell, while each corner atom contributes $\left(\frac{1}{8}\right)^{th}$ of its content. Therefore, net atoms inside a unit cell is equal to $6 \times \left(\frac{1}{2}\right) + 8 \times \left(\frac{1}{8}\right) = 4$

Coordination number:

In FCC cell, each corner atom is in contact with face centre atom. It would be simultaneously touching 4 atoms in the xy-plane,4 atoms in the yz-plane,4 atoms in zx-plane, making up a total of 12 atoms.

Coordination Number = 4+ 4 + 4 = 12

Atomic Packing Fraction:

$$APF = \frac{Zv}{V}$$

For FCC cell, Z = 4

$v = \frac{4}{3}\pi r^3$ is the Volume of spherical atom

V = volume of the unit cell= $(a)^3 = \left(2\sqrt{2}r\right)^3$

$$APF = \frac{\left[4 \times \left(\frac{4}{3}\pi r^3\right)\right]}{16\sqrt{2}r^3} = \frac{\pi}{3\sqrt{2}} = 0.74$$

Percentage void space:

$$\%\,Void\ space = (1 - APF) \times 100$$

$$\%\,Void\ space = (1 - 0.74) \times 100 = 26\%$$

Density:-

$$\rho = \frac{ZM}{N_A a^3}$$

For FCC cell Z =4 Hence,

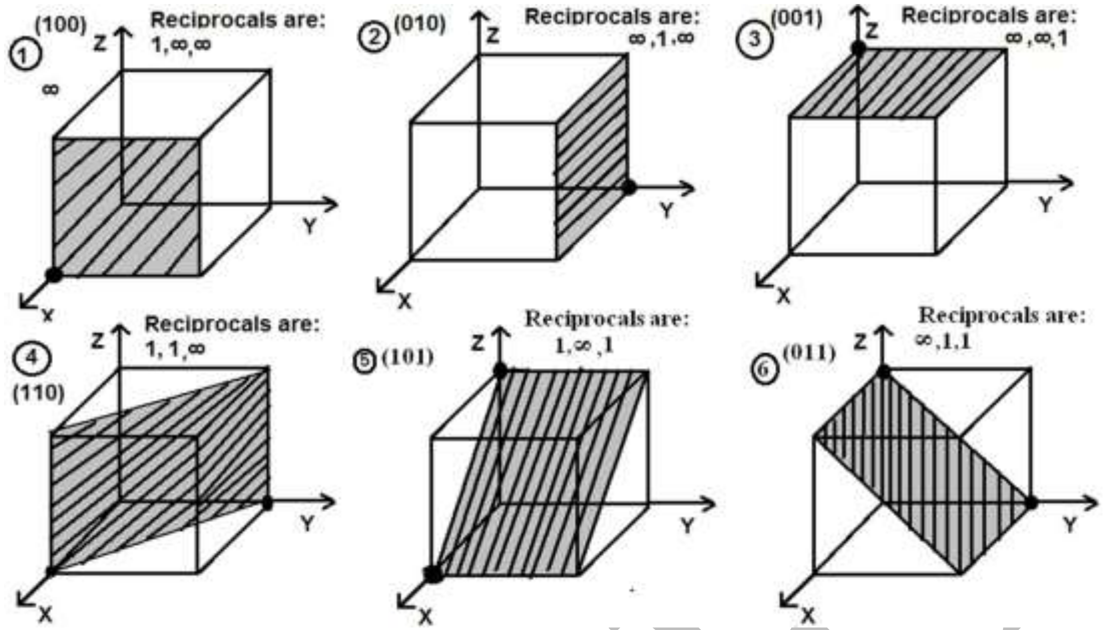
$$\rho = \frac{4M}{N_A a^3}$$

Miller indices:-

Miller introduced a system to designate a plane in a crystal. He introduced a set of three numbers that specify a plane in a crystal. Indices are the numbers which describe the different planes in the crystal. The method of designating a plane in the crystal by this set of three numbers (hkl) is known as Miller Indices.

Definition: Miller Indices are the reciprocals of the intercepts, made by the plane on the crystallographic axes, when reduced to smallest integers.

To draw various planes in a simple cubic structure:-



Procedure for Finding the Miller Indices:

Step 1: Identify the intercepts of planes along three crystallographic axes (The coordinates of the points of interception are expressed as integral multiples of the axial lengths in the respective directions)

Step 2: Determine the reciprocals of the integers

Step 3: Reduce the reciprocals to the smallest set of integers *h, k, l* by taking LCM

Step 4: The integers are written as (hkl) by enclosing in a parenthesis.

Example:

Find the Miller indices for the given plane ABC

The plane ABC has intercepts of 2a on x-axis, 2b on y-axis and 1c on z axis.

Step 1: The intercepts are 2 , 2 , 1

Step 2: The reciprocals of the intercepts are $\frac{1}{2}, \frac{1}{2}, 1$

Step 3: Taking LCM we get 1, 1, 2

Step 4: The miller Indices of the planes are (1 1 2)

Inter-planar distance:-

Crystal can be considered as arrangement of equidistance planes on which the lattice points. Interspacing distance (separation) between two planes can be calculated and significant parameter in study of crystal diffraction. Consider a set of parallel planes with indices (h k l) and among these planes, one plane is passing through origin O.

The next plane lies just parallel to first plane and gives intercepts $\frac{a}{h}, \frac{b}{k}, \frac{c}{l}$ on x, y, z axis. a, b, c are lattice vectors of a crystal. ABC is plane which intercepts x, y, z axis at points A, B, C respectively.

If we draw a perpendicular ON from O to plane ABC then ON=d. Let us consider perpendicular ON makes angle $\alpha', \ \beta' \ \gamma'$ with x, y, z axis respectively then we can consider a OAN as shown

$$cos^2\alpha' + cos^2\beta' + cos^2\gamma' = 1$$

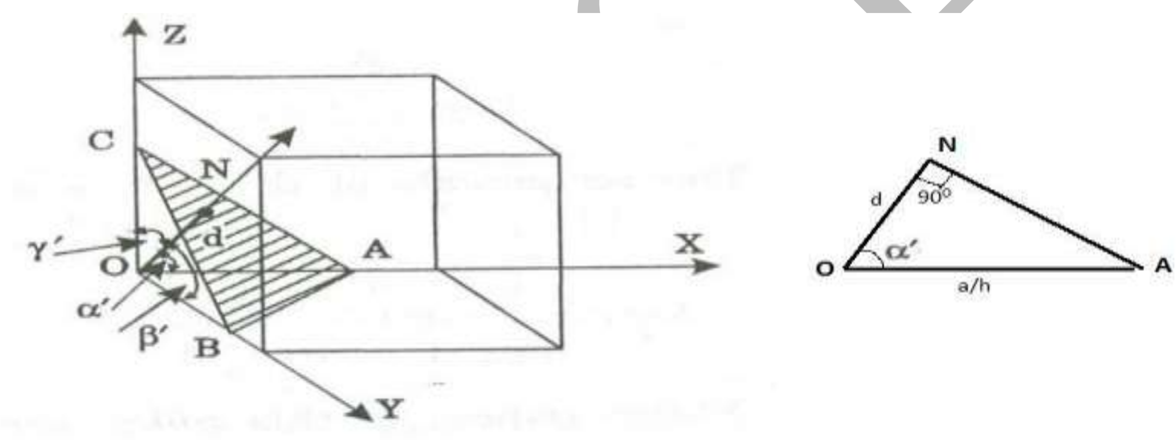


Fig: 1.18 Separation between Lattice Planes in a Crystal

According to cosine law

$$\boldsymbol{d} = \frac{\mathbf{1}}{\sqrt{\left(\frac{\boldsymbol{h}}{\boldsymbol{a}}\right)^2 + \left(\frac{\boldsymbol{k}}{\boldsymbol{b}}\right)^2 + \left(\frac{\boldsymbol{l}}{\boldsymbol{c}}\right)^2}}$$

$$\boldsymbol{cos}\gamma' = \frac{ON}{CO} = \frac{d}{c/l} = \frac{ld}{c}$$

$$\boldsymbol{cos}\alpha' = \frac{ON}{AO} = \frac{d}{a/h} = \frac{hd}{a}$$

$$\boldsymbol{cos}\beta' = \frac{ON}{BO} = \frac{d}{b/_k} = \frac{kd}{b}$$

$$\left(\frac{\boldsymbol{hd}}{\boldsymbol{a}}\right)^{\boldsymbol{2}} + \left(\frac{\boldsymbol{kd}}{\boldsymbol{b}}\right)^{\boldsymbol{2}} + \left(\frac{\boldsymbol{ld}}{\boldsymbol{c}}\right)^{\boldsymbol{2}} = \boldsymbol{1}$$

Nano Materials & Characterization

Concept of nano materials:-

Nano materials are defined as a set of substances where at least one dimension is less than approximately 100 nano metres.

The word "nano" is derived from a Greek word meaning dwarf or extremely small and means a billionth () part of a unit. A nanometer or nm is one thousand millionth of a metre, i.e., 1 nm = 10^{-9} m = 10^{-3} μm = 10 Å. In order to have an idea about nanosize, a human hair thickness accommodates around 50000 nanometers.

Examples:

- Carbon nanotubes (CNTs)
- Quantum dots (CdSe, PbS)
- Gold nanoparticles (AuNPs)
- ZnO, TiO_2 nanostructures.

Surface area to volume ratio:

Nanomaterials have a relatively larger surface area when compared to the same mass of material produced in a bulk. When a bulk material is reduced to the particle of smaller size, then the surface to volume ratio becomes very high. As the surface of the particle is involved in chemical reactions, the larger surface area can make materials more reactive.

UNIT III

QUANTUM COMPUTING

Introduction:-

Quantum computing uses the principles of quantum mechanics to build computers that perform certain types of calculations more quickly than classical computers.in quantum computing, the information is encoded in quantum system such as atoms, ions or quantum dots.

Richard Feynman proposed quantum computing in the 1980s, Arguing that quantum computers can simulate physical systems more efficiently than classical computers. The use quantum computing has enormous potential across a variety of fields, including cryptography, optimization problems, artificial intelligence, security and drug discovery.

Linear algebra for quantum computation: -

linear algebra is a branch of mathematics used to represent and manipulate quantum states(vector), quantum operations(matrices) and predict the behaviour of quantum systems. In simple words quantum computation is about performing linear algebra on quantum states.

Linear algebra deals with vectors, matrices and linear transformation.it provides a powerful tool set for solving system of linear equations, finding eigen values and eigen vectors, and manipulating vector spaces. With reference to quantum computing, linear algebra plays a vital role in representing the states of quantum systems, which are described by complex vectors in a high dimensional Hilbert space.

The principle of linear algebra enables to manipulate the vectors using operations such as rotations, reflections and entanglement, which are essential for implementing quantum algorithms.

Vectors and State Representation

Quantum States

A quantum state (qubit) is represented as a column vector in a complex vector space (Hilbert space).

Example for one qubit:

$|\psi\rangle=\alpha|0\rangle+\beta|1\rangle=\begin{bmatrix}\alpha\\ \beta\end{bmatrix}$, where $\alpha,\beta \in C$ (complex numbers)and $|\alpha|^2+|\beta|^2=1$

Basis Vectors

Standard basis (computational basis):

$|0\rangle=\begin{bmatrix}1\\0\end{bmatrix}$, $|1\rangle=\begin{bmatrix}0\\1\end{bmatrix}$

Eigenvalues and Eigenvectors

For operator A,

$A|\psi\rangle=\lambda|\psi\rangle$ where
$\lambda \rightarrow$ eigenvalue,
$|\psi\rangle \rightarrow$ eigenvector.

Linear Operators and Matrices

Quantum operations are linear transformations represented by matrices acting on vectors. If $|\psi\rangle$ is a state and Ais an operator:

$|\psi'\rangle=A|\psi\rangle$

Tensor Product (Multiple Qubits)

Used to describe multi-qubit systems.

$|a\rangle\otimes|b\rangle=|ab\rangle$

Example:

$$|0\rangle\otimes|1\rangle=|01\rangle=\begin{bmatrix}0\\1\\0\\0\end{bmatrix}$$

This forms a 4-dimensional vector space C^4 for two qubits.

Comparison of Classical and Quantum Computing

Classical computing relies on principles of Boolean algebra. Data must be processed in an exclusive binary state at any point in time; either 0 (off / false) or 1 (on / true). The millions of transistors and capacitors at the heart of computers can only be in one state at any point. In addition, there is still a limit as to how quickly these devices can be made to switch states. As we progress to smaller and faster circuits, we begin to reach the physical limits of materials and the limitations for classical laws of physics to apply

The quantum computer operates with a two-mode logic gate. In a quantum computer, a number of elemental particles such as electrons or photons can be used. Each particle is given a charge or polarization, acting as a representation of 0 and/or 1. Each particle is called a *quantum bit*, or *qubit*. The nature and behaviour of these particles form the basis of quantum computing. The two most relevant aspects of quantum physics are the principles of superposition and entanglement.

Differences between classical and quantum computing

Comparison key	Classical computer	Quantum computer
Basis of computing	Large scale multipurpose computer based on classical physics	High speed computer based on quantum mechanics
Information storage	Bit-based information storage using voltage/charge	Quantum bit-based information storage using electron spin or polarization
Bit values	Bits having a value of either 0 or 1 can have a single value at any instant	Qubits have a value of 0, 1 or sometimes linear combination of both, (a property known as superposition)
Number of possible states	The number of possible states is 2 which is either 0 or 1	The number of possible states is infinite since it can hold combinations of 0 or 1 along with some complex information
Output	Deterministic (repetition of computation on the same input gives the same output)	Probabilistic (repetition of computation on superposed states gives probabilistic answer)
Gates used for processing	Logic gates (AND, OR, NOT, etc.)	Quantum gates (X, Y, Z, H, CNOT etc.)
Operations	Operations use Boolean Algebra	Operations use linear algebra and are represented with unitary matrices
Circuit implementation	Circuit implemented in macroscopic technologies (e.g. CMOS) that are fast and scalable	Circuits implemented in microscopic technologies (e.g. nuclear magnetic resonance) that are slow and delicate

Concept of qubit and its properties

From bits to qubit

Bit: A digital computer stores and processes information using bits, which can be either 0 or 1. Physically, a bit can be anything that has two distinct configurations: one represented by "0", and the other represented by "1". It could be a light bulb that is on or off, a coin that is heads or tails, or any other system with two distinct and distinguishable possibilities. In modern computing and communications, bits are represented by the absence or presence of an electrical signal, encoding "0" and "1" respectively

Qubit is the physical carrier of quantum information. It is the quantum version of a bit, and its quantum state can be written in terms of two levels, labelled $|0\rangle$ and $|1\rangle$. $|\,\rangle$ this notation is known as 'ket' notation and $\langle\,|$ is known as 'brac' notation. Both are together called as **Dirac notations** 'Ket' is analogous to a column vector.

They are also called basis vectors and represented by two-dimensional column vectors as follows

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix} \qquad |1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$$

The qubit can be in any one of the two states as well as in the superposed state simultaneously

In quantum computation two distinguishable states of a system are needed to represent a bit of data. For example, two states of an electron orbiting a single atom is shown in the following figure. Spin up is taken as $|1\rangle$ and spin down is taken as $|0\rangle$. Similarly ground state energy level is $|0\rangle$ and excited state level is $|1\rangle$

Qubit represented by two electronic levels in an atom

This is the abstract notion of a qubit. The quantum computers actually use a physical type of qubit called a *superconducting qubit* is made from superconducting materials (of course, there are other approaches also to build the qubits)

NOTE:
In quantum computing the vectors are members of complex vector space#. Each member of this space is represented as column vector of *n* dimensions with single 1 at the location corresponding to a particular basis vector.

It is as follows

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \; |1\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \; |2\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix} \; \ldots \ldots \; |N-1\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

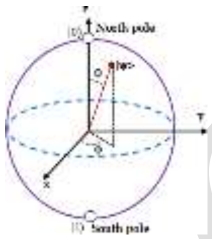
Complex vector space is a vector space which contains complex numbers

After studying the physics of qubits it is now time to look at the mathematics of qubits. Let us start with the representation of qubit using Bloch sphere in a vector space. Later on we proceed towards single qubit, multi qubit, tensor operation, operators and matrix representation

NOTE: Vector space is a set of elements (or vectors) which are added together or multiplied by real numbers. Addition of two vectors or multiplication of a vector by a scalar is satisfied here. It should not be confused with vector field

Bloch sphere

Bloch sphere is an imaginary sphere which is used to represent pure single-qubit states as a point on its surface. It has unit radius. Its North Pole and South Pole are selected to represent the basis states namely $|0\rangle$ and $|1\rangle$. North Pole represents $|0\rangle$ (say spin up↑) and South Pole represents $|1\rangle$ (say spin down ↓). All other points on the sphere represent superposed states (ie, state space). Bloch sphere allows the state of a qubit to be represented in spherical coordinates (ie, r, θ and ϕ). It is as follows



The state qubit $|\psi\rangle$ on the Bloch sphere makes an angle θ with z-axis and its projection (azimuth) makes angle ϕ with x-axis as shown. It is clear from the fig that $0 < \theta < \pi$ and $0 < \phi < 2\pi$. $|\psi\rangle$ is represented as

$$|\psi\rangle = \alpha\,|0\rangle + \beta|1\rangle$$

It can be proved that

$$|\boldsymbol{\Psi}\rangle = cos\frac{\theta}{2}|0\rangle + e^{i\phi}sin\frac{\theta}{2}|1\rangle --- (1)$$

Using this equation we can represent $|\psi\rangle$ for different θ and ϕ as follows

Case-1: let $\theta = 0$ and $\phi = 0$, then eq (1) becomes

$$|\boldsymbol{\Psi}\rangle = cos\,0|0\rangle + e^{i0}sin0|1\rangle = |0\rangle + 0$$

$$\therefore |\boldsymbol{\Psi}\rangle = |0\rangle$$

Case-2: let $\theta = \pi$ and $\phi = 0$, then eq (1) becomes

$$|\boldsymbol{\Psi}\rangle = cos\,\frac{\pi}{2}|0\rangle + e^{i0}sin\frac{\pi}{2}|1\rangle = 0 + |1\rangle$$

$$\therefore |\boldsymbol{\Psi}\rangle = |1\rangle$$

Case-3: let $\theta = \pi/2$ and $\phi = 0$, then eq (1) becomes

$$|\Psi\rangle = cos\,\frac{\pi}{4}|0\rangle + e^{i0}sin\frac{\pi}{4}|1\rangle$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|\Psi\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

Case-4: let $\theta = \pi/2$ and $\phi = \pi$, then eq (1) becomes

$$|\Psi\rangle = cos\,\frac{\pi}{4}|0\rangle + e^{i\pi}sin\frac{\pi}{4}|1\rangle$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

$$|\Psi\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

In the above discussion we have represented only single qubit state. Bloch sphere is a nice visualization of single qubit states.

Multiple Qubits

Single qubits are interesting, but individually they offer no computational advantage. We will now look at how to represent multiple qubits, and how these qubits can interact with each other.

Two qubits

Consider two qubits. They can be in any one of four possible states represented as $|00\rangle$, $|01\rangle$, $|10\rangle$ and $|11\rangle$. The interaction among these qubits is described by creating a new vector space[2] using a special kind of operation called *tensor product* or *Kronecker product*. It is as follows

Let U and V are two 2-d vectors given as

$$U = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}, V = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

Their tensor product is

$$U \otimes V = \begin{bmatrix} x_1 \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \\ y_1 \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \end{bmatrix}$$

[2] Vector space is a set of elements (or vectors) which are added together or multiplied by real numbers or scalars. Addition of two vectors or multiplication of a vector by a scalar is satisfied here. It should not be confused with vector field

$$U \otimes V = \begin{bmatrix} x_1x_2 \\ x_1y_2 \\ y_1x_2 \\ y_1y_2 \end{bmatrix}$$

Based on this we can write |00⟩ as follows

$$|0\rangle \otimes |0\rangle = \begin{bmatrix} 1\begin{bmatrix}1\\0\end{bmatrix} \\ 0\begin{bmatrix}1\\0\end{bmatrix} \end{bmatrix} = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$

Sometimes we avoid the symbol ⊗ and write directly as

$$|00\rangle = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$

Similarly

$$|01\rangle = \begin{bmatrix}0\\1\\0\\0\end{bmatrix} \quad |10\rangle = \begin{bmatrix}0\\0\\1\\0\end{bmatrix} \text{ and } |11\rangle = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

The state qubit is (ie, linear combination of these four)

$$|\Psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

For 2 qubit system we have 4 complex amplitudes namely $\alpha_{00},$ $\alpha_{01},$ α_{10} and $\alpha_{11}.$ According to normalization condition

$$|\alpha_{00}|^2 + |\alpha_{01}|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 = 1$$

Similarly if there are 3 qubits there will be 8 complex amplitudes and in general for *n* qubits we will have 2^n complex amplitudes. This means that a basis state is represented by a number 0 to 2^{n-1}. The superposition state is represented as

$$|\Psi\rangle = \sum_{x=0}^{2^{n-1}} \propto_x |x\rangle$$

Qubit has two quantum states
two states as well as in the superposed state simultaneously.

Dirac representation and Matrix operations

Matrix representation of 0 and 1 states

In Quantum mechanics, Brac-Ket notation is a standard notation for describing quantum states. The notation | ⟩ is known as 'ket' notation and ⟨ | is known as 'brac' notation. Both are together called as **Dirac notations.**

The 'ket' vector typically represented as a column vector and 'brac' vector typically represented as a row vector as follows

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} ------ \text{ket notations}$$

$$\langle 0| = [1 \quad 0] \qquad \langle 1| = [0 \quad 1] ------ \text{brac notations}$$

Hence, any arbitrary state can be represented as

$$|\psi> = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \qquad |\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

Some of the properties of these notations are

i. Addition of two kets gives another ket (commutative)

$$|A\rangle + |B\rangle = |C\rangle = |B\rangle + |A\rangle$$

ii. Addition of kets obeys associative property

$$|A\rangle + (|B\rangle + |C\rangle) = (|A\rangle + |B\rangle) + |C\rangle$$

iii. If c_1 and c_2 are scalars or a complex numbers and $|A\rangle$ is a ket then

$$(c_1 + c_2)|A\rangle = c_1|A\rangle + c_2|A\rangle$$

iv. In a complex vector space for every ket there is unique brac. Brac is the Hermitian# conjugate of the ket.

$$\text{If } |A\rangle = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \text{ then } \langle A| = [A_1^* \quad A_2^*]$$

v. Bracs are useful in calculating probability amplitudes.
For ex, the probability amplitude of $|1\rangle$ is β which can be calculated as follows

$$\langle 1||\boldsymbol{\Psi}\rangle = \langle 1|\alpha\,|0\rangle + \langle 1|\beta\,|1\rangle$$
$$\langle 1||\boldsymbol{\Psi}\rangle = \alpha\langle 1|\,|0\rangle + \beta\,\langle 1||1\rangle$$
$$\langle 1||\boldsymbol{\Psi}\rangle = \alpha[0 \quad 1]\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta[0 \quad 1]\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$\langle 1|\boldsymbol{\Psi}\rangle = \alpha \times 0 + \beta \times 1$$
$$\langle 1|\boldsymbol{\Psi}\rangle = \beta$$

Similarly

$$\langle 0|\boldsymbol{\Psi}\rangle = \alpha$$

vi. If $\langle U|$ and $\langle V|$ are two bracs then

Hilbert space

A Hilbert space is a fundamental concept in mathematics, particularly in functional analysis and quantum mechanics. It is a complete inner product space, which means it is a vector space equipped with an inner product and satisfies the following properties:

1. **Vector space**: A Hilbert space is a set of vectors that can be added together and multiplied by scalars (real or complex numbers). These operations satisfy the usual axioms of a vector space, such as associativity, commutativity, and distributivity.
2. **Inner product:** The space is equipped with an inner product, which is a function that takes two vectors and returns a scalar, satisfying properties like linearity, conjugate symmetry, and positive definiteness. The inner product gives a way to measure lengths (norms) and angles between vectors.
3. **Norm**: The norm of a vector in a Hilbert space is defined as the square root of the inner product of the vector with itself. This norm induces a distance function, turning the space into a metric space.
4. **Completeness:** A Hilbert space is complete, meaning that every Cauchy sequence (a sequence of vectors where the vectors get arbitrarily close to each other) has a limit in the space. This property is crucial in mathematical analysis, ensuring that the space is large enough to accommodate all limit processes that might occur in analysis.
5. **Infinite-dimensional**: While finite-dimensional vector spaces can also be Hilbert spaces the concept is often applied to infinite-dimensional spaces, where it plays a key role in quantum mechanics and functional analysis.

Concept of quantum computer

Quantum computing is a rapidly-emerging field focused on the development of computer technologies centered on the principles of Quantum Physics. Quantum Physics explains the nature and behaviour of energy and matter on the quantum (atomic and subatomic) scale. Elementary particles such as protons, neutrons and electrons can exist in two or more states at a time. This fundamental behaviour is utilized in designing the quantum computation processing units and in fact it is more efficient than classical computation

Quantum computing uses a combination of bits to perform specific computational tasks with greater efficiency than their classical counterparts. Even though quantum computers are not going to replace classical computers, quantum technology is significantly changing the way the world operates. The quantum computer gains much of its processing power through the ability for bits to be in multiple states simultaneously. They can perform tasks using a combination of 1's, 0's and both 1 and 0 at a time.

The core concepts of quantum computing include:

- Qubits: The basic unit of quantum information, analogous to classical bits but can exist in a superposition of states (both 0 and 1 simultaneously), enabling quantum parallelism.
- Superposition: A quantum principle where a qubit can be in multiple states at once, allowing quantum computers to process many possibilities simultaneously.
- Entanglement: A phenomenon where qubits become linked such that the state of one qubit instantaneously influences the state of another, no matter the distance, enabling complex correlations that speed up computation.
- Quantum gates and algorithms: Operations that manipulate qubit states in quantum circuits to perform calculations that can outperform classical computers in certain tasks, such as factoring large numbers or simulating molecular structures.

Quantum computing systems for information processing:-

Quantum computing systems represent a transformative approach to information processing, leveraging the principles of quantum mechanics to solve problems that are intractable for classical computers. Quantum algorithms use qubits instead of classical bits.

Key features include:

Superposition: A qubit can be in a combination of states, enabling parallel computation.

Entanglement: Qubits can be correlated in ways that classical bits cannot.

Interference: Quantum algorithms use interference to amplify correct outcomes.

These properties allow quantum systems to process information in fundamentally new ways.

Evolution of quantum systems:-

1. 1982: Richard Feynman lectures on the potential advantages of computing with quantum systems.
2. 1985: David Deutsh publishes the idea of a "universal quantum computer"
3. 1994: Peter Shor algorithm that can find factors of large numbers.
4. 1996: Grove's search algorithm
5. 1996: Seth Lloyd proposes a quantum algorithm which can simulate quantum-mechanical systems
6. 1999: D-Wave Systems founded by Geordie Rose
7. 2000: Eddie Farhi at MIT develops idea for adiabatic quantum computing
8. 2001: IBM and Stanford University publish the first implementation of Shor's algorithm, factoring 15 into its prime factors on a 7-qubit processor.
9. 2010: D-Wave One: first commercial quantum computer released (annealer)
10. 2016: IBM makes quantum computing available on IBM Cloud
11. 2019: Google's Quantum Supremacy

(i) Emergence of Quantum Computing (1980-1994): Richard Feynman, Paul Benioff, and David Deutsch demonstrated the potential advantages of harnessing quantum mechanics for computation. The development of quantum logic gates and the concept of a universal quantum computer advanced quantum computing research.

(ii) Development of Quantum Algorithms (1994-2000): The mid-1990s saw significant progress in the development of quantum algorithms. The early quantum algorithms of Shor's and Grover's generated significant interest and hence the research in quantum computing, i.e. error-correcting codes, and hardware implementations accelerated.

(iii) Race to Build Quantum Computers (2000-2021): In the early 21st century, researchers, tech giants, and startups competed to build practical quantum computers. The key developments in the race to build quantum computers are:

(a) Approaches to Quantum Computing: Different approaches have been explored for building quantum computers. Some of the prominent methods include Tapped Ions, Superconducting Qubits, Topological Qubits and so on.

(b) D-Wave Systems and Quantum Annealing: In 2011, a Canadian company D-Wave Systems claimed to have built a quantum computer, the D-Wave One, based on quantum annealing.

(c) Google's Quantum Supremacy: In 2019, with its 53-qubit Sycamore processor, Google achieved "quantum supremacy" in quantum computing. It took Google's quantum computer 200 seconds to solve a problem that would have taken a classical supercomputer 10,000 years. For the first time, a quantum computer outperformed classical computers in a well-defined computational task, providing a proof-of-concept for quantum computing's potential.

(d) Ongoing Challenges and Developments: These include improving qubit coherence times, developing scalable and fault-tolerant architectures, and creating practical error correction techniques. In order to address these challenges and push quantum

Quantum Measurements:-

Quantum measurement and probability are key concepts in quantum mechanics. They reveal the strange, uncertain nature of the subatomic world. The measurement process in quantum computing is probabilistic. When a qubit is measured, it collapses from its superposition state to one of the basis states, ∣0⟩|0\rangle∣0⟩ or ∣1⟩|1\rangle∣1⟩, with probabilities determined by the coefficients α and β.
This probabilistic nature contrasts with the deterministic measurement of classical bits and introduces new challenges and opportunities in quantum algorithm design. For example:

In classical computing, an algorithm would need to check each entry one by one, resulting in a linear time complexity, O(N), where N is the number of entries. O(N) signifies linear time complexity, meaning an algorithm’s execution time grows directly and proportionally to the number of entries (N) in its input.

In quantum computing (using Grover’s algorithm), a qubit-based system can search the database in $O(\sqrt{N})$ time, demonstrating a quadratic speedup due to the principles of superposition and quantum interference.
An $O(\sqrt{N})$ time complexity is a sublinear performance, meaning the algorithm’s execution time increases much slower than the input size itself. This is a relatively efficient complexity for many problems.

Uncertainty Principle and Measurement Implications

(i) Uncertainty principle arises from the nature of quantum measurement.
(ii) Demonstrates incompatibility of certain observables.
(iii) Limits precision with which certain pairs of physical properties can be known simultaneously; position and momentum cannot be measured with arbitrary precision at the same time.
(iv) Heisenberg’s microscope thought experiment illustrates the uncertainty principle in action.
(v) Impacts various fields of physics such as atomic spectroscopy, quantum optics.

Advantages of Quantum Computing over Classical Computation:-

1. Quantum bits (qubits) can exist in multiple states (0 and 1) simultaneously. This allows quantum computers to process a vast number of possibilities at once.
2. Quantum algorithms like Shor's algorithm (for factoring large numbers) and Grover's algorithm (for searching unsorted databases) can outperform the best classical algorithms exponentially or quadratically.
3. Quantum computers can naturally simulate molecular and atomic systems, which is extremely hard for classical computers. Useful in drug discovery, materials science, and chemistry.
4. Quantum key distribution (QKD) enables secure communication based on quantum principles, which is nearly impossible to hack without detection.
5. Quantum algorithms can potentially solve optimization problems (e.g., logistics, finance, AI training) much faster than classical approaches.

Challenges of Quantum Computing over Classical Computation

As of now there are some technical difficulties and limitations in building quantum computers. Some of them are

1. As the number of quantum gates in a network increases, more interacting qubits are involved, and it is very difficult to monitor their interactions
2. The surrounding environment will affect the interactions of qubits (both superposition and entanglement). As a result the quantum information will spread outside the quantum computer and be lost into the environment, thus spoiling the computation. This process is called ***de- coherence.*** How long quantum information will survive before it is spread out is known as ***de- coherency time***
3. The number of operations that can be performed before the information is lost due to de- coherency is therefore limited.
4. Quantum chips must be kept at very low temperature to create super positions and entanglement of qubits
5. The final output of the quantum computers is in the form of a probability. When the question is repeated, the answer changes. Hence repeated operations are required to get correct answer.

There are different types of quantum gates. *Single-qubit gates* can flip a qubit from 0 to 1 as well as allowing superposition states to be created. *Two-qubit gates* allow the qubits to interact with each other and can be used to create **quantum entanglement** (a strange phenomenon that can't be explained by classical physics).

Some of the important single qubit gates are discussed here. They all are represented by 2 × 2 matrix. (Note that X, Y and Z gates are already discussed earlier under the heading Pauli's matrices. So, it is a sort of repetition)

Single qubit gates

1. X – Gate

This is also called as Pauli X – gate. It is given by

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

When X operates on |0⟩ and |1⟩ the output will be inverted (ie, |0⟩ becomes |1⟩ and |1⟩ becomes|0⟩)

$$X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$$

$$X|1\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

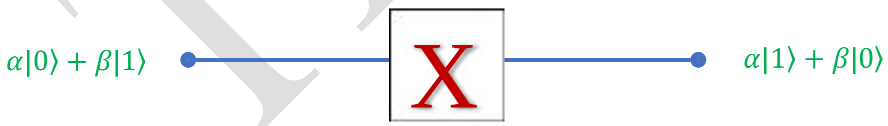
Since X inverts each input it is also called as *bit-flip* gate. If a superposed qubit goes through X gate, the result will be

$$X|\mathbf{\Psi}\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \beta \\ \alpha \end{bmatrix} = \alpha|1\rangle + \beta|0\rangle$$

So,

$$X|\mathbf{\Psi}\rangle = \alpha|1\rangle + \beta|0\rangle$$

Symbolically these gates are represented as follows



2. Y – Gate

This is also called as Pauli Y – gate. It is given by

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

When Y operates on |0⟩ and |1⟩

$$Y|0\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0\times 1 + (-i)\times 0 \\ i\times 1 + 0\times 0 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix} = i\begin{bmatrix} 0 \\ 1 \end{bmatrix} = i|1\rangle$$

$$Y|1\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0\times 0 + (-i)\times 1 \\ i\times 0 + 0\times 1 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix} = -i\begin{bmatrix} 1 \\ 0 \end{bmatrix} = -i|0\rangle$$

So,

$Y|0\rangle = i|1\rangle$ and $Y|1\rangle = -i|0\rangle$

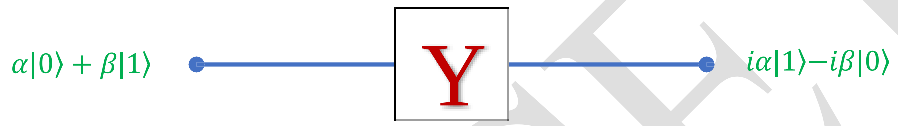
If a superposed qubit goes through Y gate, the result will be

$$Y|\mathbf{\Psi}\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 0\times\alpha + (-i)\times\beta \\ i\times\alpha + 0\times\beta \end{bmatrix} = \begin{bmatrix} -i\beta \\ i\alpha \end{bmatrix} = -i\beta|0\rangle + i\alpha|1\rangle$$

So,

$$Y|\mathbf{\Psi}\rangle = i\alpha|1\rangle - i\beta|0\rangle$$

Symbolically these gates are represented as follows



3. Z – Gate

This is also called as Pauli Z – gate. It is given by

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

When Z operates on |0⟩ and |1⟩ the phase will change. Hence this is also called as *phase-flip* gate

$$Z|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1\times 1 + 0\times 0 \\ 0\times 1 + (-1)\times 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

$$Z|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1\times 0 + 0\times 1 \\ 0\times 0 + (-1)\times 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} = -\begin{bmatrix} 0 \\ 1 \end{bmatrix} = -|1\rangle$$

So,

$Z|0\rangle = |0\rangle$ and $Z|1\rangle = -|1\rangle$

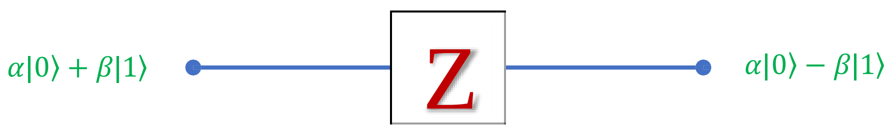
If a superposed qubit goes through Z gate, the result will be

$$Z|\mathbf{\Psi}\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 1\times\alpha + 0\times\beta \\ 0\times\alpha + (-1)\times\beta \end{bmatrix} = \begin{bmatrix} \alpha \\ -\beta \end{bmatrix} = \alpha|0\rangle - \beta|1\rangle$$

So,

$$Z|\mathbf{\Psi}\rangle = \alpha|0\rangle - \beta|1\rangle$$

Symbolically these gates are represented as follows

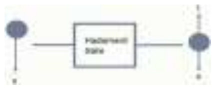


The truth tables for X, Y and Z gates are as follows

X- gate		Y-gate		Z-gate	
Input	**Output**	**Input**	**Output**	**Input**	**Output**
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$i\|1\rangle$	$\|0\rangle$	$\|0\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$-i\|0\rangle$	$\|1\rangle$	$-\|1\rangle$
$\alpha\|0\rangle + \beta\|1\rangle$	$\alpha\|1\rangle + \beta\|0\rangle$	$\alpha\|0\rangle + \beta\|1\rangle$	$i\alpha\|1\rangle - i\beta\|0\rangle$	$\alpha\|0\rangle + \beta\|1\rangle$	$\alpha\|0\rangle - \beta\|1\rangle$

4. Hadamard Gate – The gate to superposition

The Hadamard Gate is a well-known gate that brings a qubit into a superposition state. Similar to the Pauli-X gate, the Hadamard Gate acts on a single qubit, and can be represented by a 2 x 2 matrix as follows



Hadamard gate brings a qubit in superposition

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Let us find out what happens when Hadamard gate operates on a qubit that is in the $|0\rangle$ state.

$$H|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \times 1 + 1 \times 0 \\ 1 \times 1 + -1 \times 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$H|0\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \quad --- (1)$$

Let us find out what happens when Hadamard gate operates on a qubit that is in the $|1\rangle$ state.

$$H|1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \times 0 + 1 \times 1 \\ 1 \times 0 + -1 \times 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$H|1\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} --- (2)$$

If a superposed qubit goes through H gate, the result will be

$$H|\psi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \times \alpha + 1 \times \beta \\ 1 \times \alpha + -1 \times \beta \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} \alpha + \beta \\ \alpha - \beta \end{bmatrix}$$

$$H|\psi\rangle = \frac{\alpha+\beta}{\sqrt{2}}|0\rangle + \frac{\alpha-\beta}{\sqrt{2}}|1\rangle$$

$$H|\psi\rangle = \alpha\frac{|0\rangle+|1\rangle}{\sqrt{2}} + \beta\frac{|0\rangle-|1\rangle}{\sqrt{2}} - - - (3)$$

The above equations shows that, after applying the Hadamard gate to a qubit that are in $|0\rangle$ & $|1\rangle$ states enter a new superposed states. This is the major difference between X, Y, Z and H gates. In X, Y and Z gates we get only single state whereas in H gate we get superposed state.
The probability of measuring 0 and 1 is

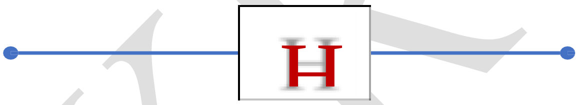
$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$$

The truth table is as follows

INPUT	OUTPUT
$\|0\rangle$	$\frac{\|0\rangle+\|1\rangle}{\sqrt{2}}$
$\|1\rangle$	$\frac{\|0\rangle-\|1\rangle}{\sqrt{2}}$
$\|\psi\rangle = \alpha\|0\rangle + \beta\|1\rangle$	$\alpha\frac{\|0\rangle+\|1\rangle}{\sqrt{2}} + \beta\frac{\|0\rangle-\|1\rangle}{\sqrt{2}}$

The circuit symbol is as follows

$\alpha|0\rangle + \beta|1\rangle$



$\alpha\frac{|0\rangle+|1\rangle}{\sqrt{2}} + \beta\frac{|0\rangle-|1\rangle}{\sqrt{2}}$

5. Phase Gate (S Gate)

The Phase gate or S gate is a gate that transfers $|0\rangle$ into $|0\rangle$ and $|1\rangle$ into $i|1\rangle$. It is represented as

$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

If we apply S gate to a state $|0\rangle$ it will remain same

$$S|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1(1)+0(0) \\ 0(1)+i(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$S|0\rangle = |0\rangle$$

If we apply S gate to a state $|1\rangle$ it will be transformed into $i|1\rangle$

$$S|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1(0)+0(1) \\ 0(0)+i(1) \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix} = i\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$S|1\rangle = i|1\rangle$$

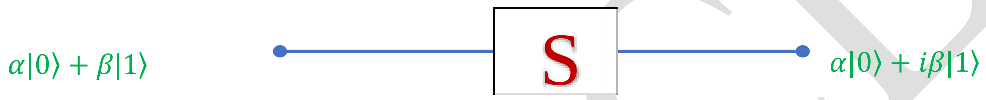
It transforms the state $\alpha|0\rangle + \beta|1\rangle$ to the state $\alpha|0\rangle + i\beta|1\rangle$

$$S|\psi\rangle = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \alpha \\ i\beta \end{bmatrix}$$

The truth table is as follows

Input	Output
$\|0\rangle$	$\|0\rangle$
$\|1\rangle$	$i\|1\rangle$
$\|\psi\rangle = \alpha\|0\rangle + \beta\|1\rangle$	$\alpha\|0\rangle + i\beta\|1\rangle$

The symbol is as follows



6. T- Gate

The T-gate is a very commonly used gate and it is given by

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{bmatrix}$$

If the input is $|0\rangle$ then the output is also $|0\rangle$

$$T|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$T|0\rangle = |0\rangle$$

If the input is $|1\rangle$ then the output state is $e^{\frac{i\pi}{4}}|1\rangle$

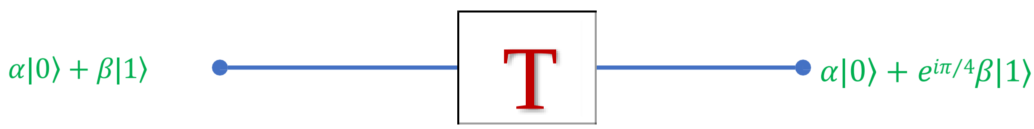
$$T|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ e^{\frac{i\pi}{4}} \end{bmatrix} = e^{\frac{i\pi}{4}} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$T|1\rangle = e^{\frac{i\pi}{4}}|1\rangle$$

It transforms the state $\alpha|0\rangle + \beta|1\rangle$ to $\alpha|0\rangle + e^{i\pi/4}\beta|1\rangle$

$$T|\psi\rangle = \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta e^{\frac{i\pi}{4}} \end{bmatrix}$$

The following figure shows quantum T- gate and the table gives the truth table.



The truth table is as follows

Input	Output
$\|0\rangle$	$\|0\rangle$
$\|1\rangle$	$e^{\frac{i\pi}{4}}\|1\rangle$
$\alpha\|0\rangle + \beta\|1\rangle$	$\alpha\|0\rangle + e^{i\pi/4}\beta\|1\rangle$

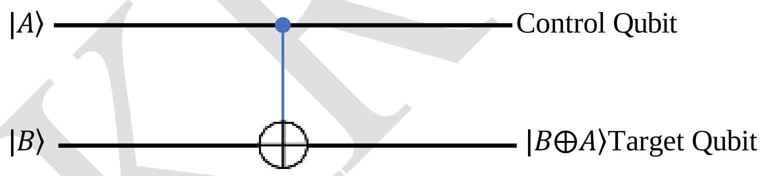
Multiple Qubit gates

As mentioned in the earlier section, Single qubits are interesting, but individually they offer less computational advantage. It is hence essential to look for multiple qubit system and the operation on them. Quantum gates operating on multiple qubits are called as *multiple qubit gates*. Some of them are as follows

1. Controlled Gate (CNOT)

The CNOT gate is a two-qubit operation, where the first qubit is referred as the ***control qubit*** (A) and the second qubit as the ***target qubit*** *(B)*. If the control qubit is $|1\rangle$ then it will flip the target qubit state from$|0\rangle$ to $|1\rangle$ or from $|1\rangle$ to $|0\rangle$. When the control qubit is in state $|0\rangle$ then the target qubit remains unchanged. In fact CNOT applies X on target whenever its control is in state $|1\rangle$

The symbolic representation is as follows. The upper line represents control qubit and bottom line represents target qubit



In the combined qubit, first term is control qubit and the second term is target qubit. For ex, in $|AB\rangle$, A is control qubit and B is target qubit

NOTE: In diagram the control qubit is represented by and target is represented by $\oplus$

Discussion for 4 different input states

1. Input state $|00\rangle$ (Control qubit = 0, Target qubit = 0): Both the bits remain unaltered. Hence, the output state is the same as the input state or $|00\rangle \rightarrow |00\rangle$
2. Input state $|01\rangle$ (Control qubit = 0, Target qubit = 1): Both the bits remain unaltered. Again, the output state is the same as the input state or $|01\rangle \rightarrow |01\rangle$
3. Input state $|10\rangle$ (Control qubit =1, Target qubit = 0): The target qubit is flipped to 1. Therefore, the output state has both qubits 1 or $|10\rangle \rightarrow |11\rangle$
4. Input state $|11\rangle$ (Control qubit =1, Target qubit = 1): The target qubit is flipped to 0. Therefore, the output state becomes $|10\rangle$ or $|11\rangle \rightarrow |10\rangle$.

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The truth table of a CNOT gate is as follows

Input	Output
$\|00\rangle$	$\|00\rangle$
$\|01\rangle$	$\|01\rangle$
$\|10\rangle$	$\|11\rangle$
$\|11\rangle$	$\|10\rangle$

We know that two qubits can be in any one of four possible states represented as $|00\rangle$ $|01\rangle|10\rangle$ and $|11\rangle$. The matrix form of them are

$$|00\rangle = \begin{bmatrix}1\\0\\0\\0\end{bmatrix} \; |01\rangle = \begin{bmatrix}0\\1\\0\\0\end{bmatrix} \; |10\rangle = \begin{bmatrix}0\\0\\1\\0\end{bmatrix} \; |11\rangle = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

The state qubit is $|\Psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$. When it is operated by CNOT we get $\text{CNOT}(\alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle) = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|11\rangle + \alpha_{11}|10\rangle$

From this we can construct the matrix form of CNOT gate as follows (it is 4 ×4 matrix)

The $|00\rangle$remains same as $|00\rangle$. Hence the first column is $\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$

The $|01\rangle$remains same as $|01\rangle$. Hence the second column is $\begin{bmatrix}0\\1\\0\\0\end{bmatrix}$

The $|10\rangle$changes to $|11\rangle$. Hence the third column changes from $\begin{bmatrix}0\\0\\1\\0\end{bmatrix}$ to $\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$

The $|11\rangle$changes to $|10\rangle$. Hence the fourth column changes from $\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$ to $\begin{bmatrix}0\\0\\1\\0\end{bmatrix}$

Hence the matrix form of CNOT gate is

$$CNOT = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 1\\0 & 0 & 1 & 0\end{bmatrix}$$

Ex (1) S.T the $|00\rangle$remains same as $|00\rangle$ when operated by CNOT

$$\text{CNOT}|00\rangle = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 1\\0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$

$$\text{CNOT}|00\rangle = \begin{bmatrix}1+0+0+0\\0+0+0+0\\0+0+0+0\\0+0+0+0\end{bmatrix} = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$

$$\therefore \text{CNOT}|00\rangle = |00\rangle$$

Ex (2) S.T the $|01\rangle$remains same as $|01\rangle$ when operated by CNOT

$$\text{CNOT}|01\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{CNOT}|01\rangle = \begin{bmatrix} 1+0+0+0 \\ 0+1+0+0 \\ 0+0+0+0 \\ 0+0+0+0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \text{CNOT}|01\rangle = |01\rangle$$

Ex (3) S.T the $|10\rangle$changes to $|11\rangle$ when operated by CNOT

$$\text{CNOT}|10\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\text{CNOT}|10\rangle = \begin{bmatrix} 0+0+0+0 \\ 0+0+0+0 \\ 0+0+0+0 \\ 0+0+1+0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore \text{CNOT}|10\rangle = |11\rangle$$

Ex (4) S.T the $|11\rangle$changes to $|10\rangle$ when operated by CNOT

$$\text{CNOT}|11\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{CNOT}|11\rangle = \begin{bmatrix} 0+0+0+0 \\ 0+0+0+0 \\ 0+0+0+1 \\ 0+0+0+0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

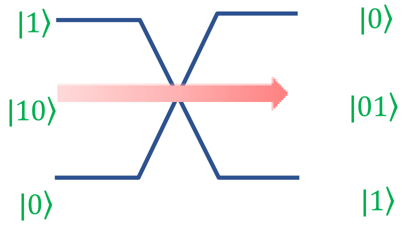
$$\therefore \text{CNOT}|11\rangle = |10\rangle$$

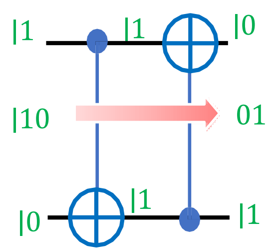
2. Swap Gate

In quantum computation sometimes we need to move state between two qubits, ie from control to target and vice versa. This is nothing but ***swapping*** of the states and the gate used for this purpose is known as SWAP gate.

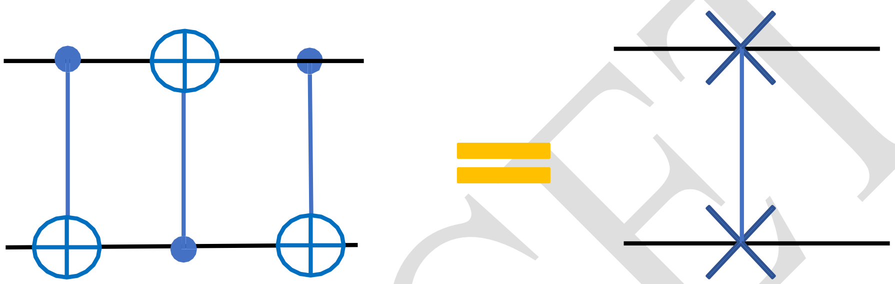
SWAP gate is a two qubit operation gate and swaps the state of the two qubits involved in the operation. It contains 3 CNOT gates.

The action of SWAP gate is explained by taking two CNOT gates as follows where $|10\rangle$ is swapped to $|01\rangle$



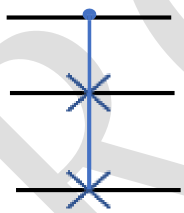


But for effective swapping of the states there must be minimum of 3 CNOT gates. The SWAP circuit



is as given below,

It is also represented as

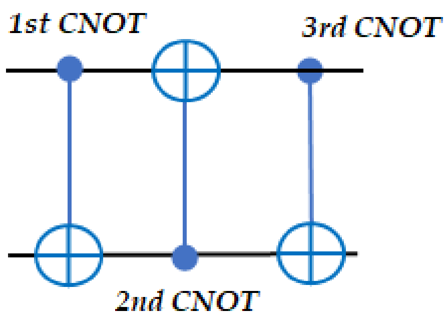


The matrix form of SWAP gate is given by

$$SWAP = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ex (1) S.T the state $|00\rangle$ remains undisturbed by the SWAP gate operation

Consider the SWAP circuit diagram



We know that in CNOT gate if the control qubit is in $|1\rangle$ state then it will flip the target qubit from $|0\rangle$ to $|1\rangle$ and vice versa (otherwise no). So, when $|00\rangle$ is given, the 1st CNOT is **not** satisfied. We stay in the state $|00\rangle$.The 2nd CNOT's control is **not** satisfied. We stay in the state $|00\rangle$.The 3rd CNOT' is also **not** satisfied. We finally stay in the state $|00\rangle$. The same can be verified using matrix analysis as follows

$$SWAP|00\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+0+0+0 \\ 0+0+0+0 \\ 0+0+0+0 \\ 0+0+0+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore SWAP|00\rangle = |00\rangle$$

Ex (2) S.T the state $|10\rangle$ is swapped to $|01\rangle$ by SWAP gate operation

$$SWAP|10\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0+0+0+0 \\ 0+0+1+0 \\ 0+0+0+0 \\ 0+0+0+0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore SWAP|10\rangle = |01\rangle$$

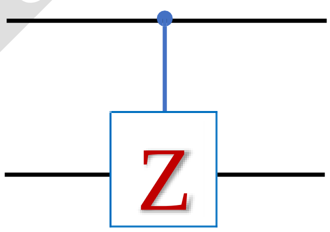
Truth table of swap gate is as follows

Input	Output
$|00\rangle$	$|00\rangle$
$|01\rangle$	$|10\rangle$
$|10\rangle$	$|01\rangle$
$|11\rangle$	$|11\rangle$

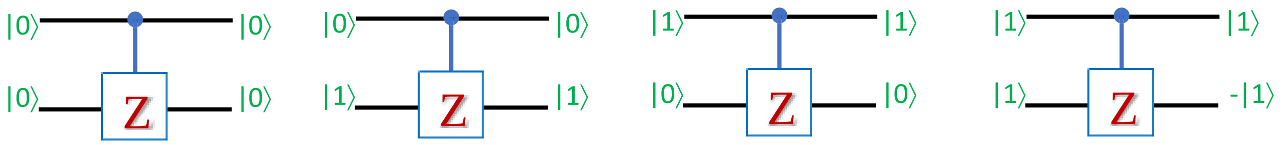
3. Controlled-Z Gate

CNOT gate can be extended in a way that it can work on two qubits based upon a single control qubit. C-Z gate is one such gate. In this gate there is one control qubit and Z unitary matrix as target qubit. If the control qubit is in state $|1\rangle$ then it acts on target Z and will flip the state (ie, there is 180^0 phase change)

The circuit is represented as follows.



Some of the examples are given below



No change because control bit is |0⟩ | *No change because control bit is |0⟩* | *Control bit acts on target but there is no flip of |0⟩* | *Control bit acts on target and flip |1⟩ to -|1⟩*

The truth table of a controlled-Z gate:

Input	**Output**
$\|00\rangle$	$\|00\rangle$
$\|01\rangle$	$\|01\rangle$
$\|10\rangle$	$\|10\rangle$
$\|11\rangle$	$-\|11\rangle$

The action of a controlled-Z gate is specified as follows

$$Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Ex (1): S.T the state $|10\rangle$ remains un affected when operated by C-Z gate

$$U_Z|10\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0+0+0+0 \\ 0+0+0+0 \\ 0+0+1+0 \\ 0+0+0+0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\therefore U_Z|10\rangle = |10\rangle$$

Ex (2): S.T the state $|11\rangle$ flips to - $|11\rangle$ when operated by C-Z gate

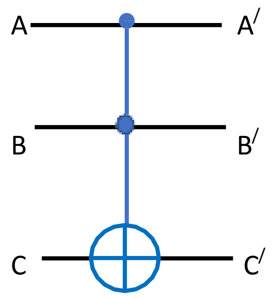
$$U_Z|11\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0+0+0+0 \\ 0+0+0+0 \\ 0+0+0+0 \\ 0+0+0-1 \end{bmatrix} = -\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore U_Z|11\rangle = -|11\rangle$$

4. Toffoli Gate

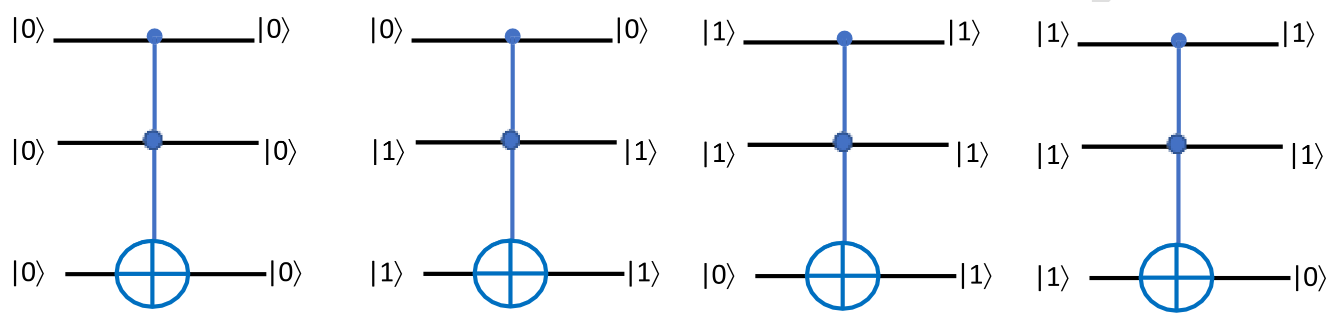
The **Toffoli gate** or **controlled-controlled-NOT (CCNOT)** gate is a logic gate having three input qubits. The first two bits are control bits which remain unaffected by the action of Toffoli Gate. The third is the target bit which is inverted (ie, changes from 0 to 1 or 1 to 0) if both the control bits are 1; else it does not change.

The circuit and the truth table are as follows



Input			Output		
A	B	C	A/	B/	C/
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0

Some examples are given here



The Toffoli gate can be expressed as an 8 by 8 matrix as follows

$$U_T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

.

NOTE:

- This is a *reversible* (no information is lost) and *universal* (all reversible logic circuits can be built using Toffoli gates).
- It can be verified that this matrix is unitary and thus the Toffoli gate is a legitimate quantum gate. The quantum Toffoli gate can be used to simulate irreversible classical logic gates and ensures that the quantum gates are capable of performing any computation that a classical computer can do

Limitations of quantum computing

As of now there are some technical difficulties and limitations in building quantum computers. Some of them are

- As the number of quantum gates in a network increases, more interacting qubits are involved, and it is very difficult to monitor their interactions

Qubit

A qubit (or quantum bit) is the quantum mechanical analogue of a classical bit.

In classical computing, the information is encoded in bits, where each bit can have the value zero or one. In quantum computing the information is encoded in qubits.

A qubit is a two-level quantum system where the two basis qubit states are usually written as $|0\rangle$, $|1\rangle$ or

(unlike a classical bit) in a linear combination of both states$|0\rangle+|1\rangle$. The name

of this phenomenon is known as **superposition**.

Examples for qubits: Single photons, neutral atoms, spin half electrons, Quantum dots, superconductors, nuclear spin set c

Entanglement

Entanglement is the ability of quantum systems to exhibit corelations between states within a superposition.

Imagine two qubits, each in the state |0> +|1> (a superposition of the 0 and1) We can entangle the two qubit such that the measurement of one qubit is always corelated to the measurement of the other qubit.

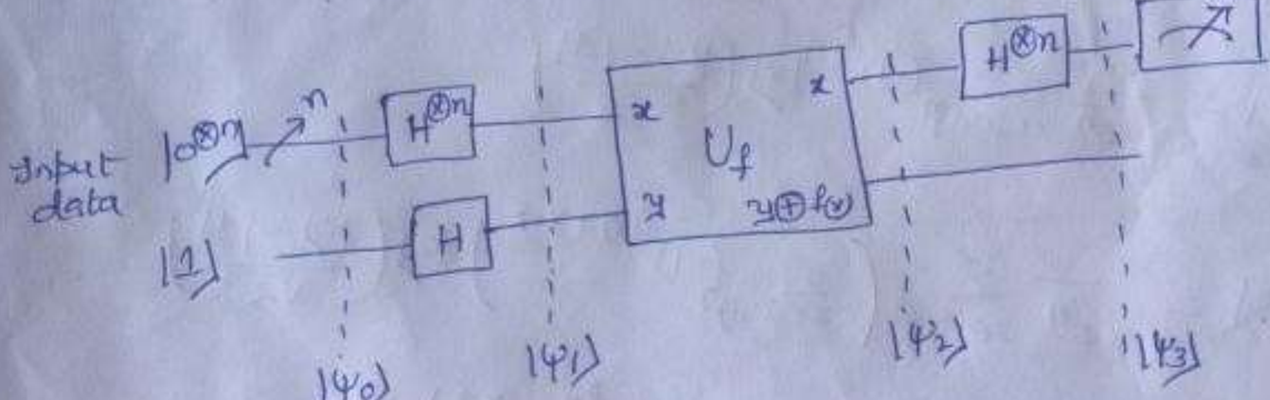
Quantum Algorithms :-

1) **Deutsch - Jozsa Algorithm :-**

We are given a hidden Boolean function f, which takes as input a string of bits, and returns either 0 or 1 for every input. That is

$$f : \{0,1\}^n \to \{0,1\}$$

The property of the given Boolean function is that it is guaranteed to either be constant or balanced. A constant function returns all 0s or all 1s for any input, while a balanced function returns 0s for half of all inputs and 1s for the other half. To find whether the function is constant or balanced.



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$$|\psi_0\rangle = |0\rangle^{\otimes n}|1\rangle$$

$$|\psi_1\rangle = \frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} |x\rangle \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$

This is a uniform superposition of all n bit strings on the data register, tensored with the ancilla in

$|-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$ state

$$|\psi_2\rangle = \frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} (-1)^{f(x)} |x\rangle \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$

Applying Quantum oracle,

$|x\rangle|y\rangle$ to $|x\rangle|y \oplus f(x)\rangle$

$$|\psi_2\rangle = \frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} |x\rangle \frac{(|f(x)\rangle - |1 \oplus f(x)\rangle)}{\sqrt{2}}$$

For 2 qubit state

$$H^{\otimes 2}|0\rangle^{\otimes 2} = \frac{1}{2}(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$$

$$= \frac{1}{2}\left[|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$$

$$= \frac{1}{2} \sum_{x \in \{0,1\}^n} |0\rangle$$

If $f(x) = 0$, $|\psi_2\rangle = \frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} |x\rangle \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$

If $f(x) = 1$, $|\psi_2\rangle = \frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} |x\rangle \left(\frac{|1\rangle - |0\rangle}{\sqrt{2}}\right)$

$$|\psi_2\rangle = \frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} (-1)^{f(x)} |x\rangle \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$

$$|\psi_3\rangle = \frac{1}{2^n} \sum_{z \in \{0,1\}^n} \left(\sum_{x \in \{0,1\}^n} (-1)^{f(x) + x \cdot z} \right) |z\rangle \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$

$$|\psi_3\rangle = \frac{1}{2^n} \sum_{z \in \{0,1\}^n} \left(\sum_{x \in \{0,1\}^n} (-1)^{f(x)} (-1)^{x \cdot z} \right) |z\rangle$$

$$|\psi_3\rangle = \frac{1}{2^n} \sum_{z \in \{0,1\}^n} \left(\sum_{x \in \{0,1\}} (-1)^{f(x) + x \cdot z} \right) |z\rangle \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right)$$

For 2 Qubit system

$H^{\otimes 2}$ means applying Hadamard to both Qubits.

1) Balanced. Assume $f(x) = 0$ for $x = 00, 01$
and $f(x) = 1$ for $x = 10, 11$
for $z = 00$

x	$f(x)$	z	$x \cdot z$	$f(x) + x \cdot z$	$(-1)^{f(x) + x \cdot z}$
00	0	00	0	0	+1
01	0	00	0	0	+1
10	1	00	0	1	-1
11	1	00	0	1	-1

2) constant $f(x) = 1$ for all x and for $z = 00$

x	$f(x)$	z	$x \cdot z$	$f(x) + x \cdot z$	$(-1)^{f(x) + x \cdot z}$
00	1	00	0	1	-1
01	1	00	0	1	-1
10	1	00	0	1	-1
11	1	00	0	1	-1

The probability of Measurement for all 0 is

$$\left| \frac{1}{2^n} \sum_{x \in \{0,1\}^n} (-1)^{f(x)} \right|^2 = \begin{cases} 1 & \text{If } f \text{ is constant} \\ 0 & \text{If } f \text{ is balanced} \end{cases}$$

2) Shor's Algorithm:-

This is to find prime factors of an Integer.
And Invented by an American mathematician
peter shor in 1994.
classically, factoring large numbers take exponential
time. shor's algorithm runs in polynomial time
using Quantum Fourier transform.
This is much faster than the most efficient known
classical factoring algorithm.

Factoring Algorithm - Number Theory

$N = p \cdot Q$ where N is an odd composite number and
p & Q are odd prime numbers

$x^2 = 1 \pmod{N}$: $x \neq \pm 1 \pmod{N}$

$x^2 = 1 \Rightarrow x^2 - 1 = 0 \pmod{N}$

N divides (x^2-1) or $(x-1)(x+1)$ but not $x-1$ and $x+1$

$GCD(N, x-1)$ and $GCD(N, x+1)$ will give factors of N

ex:

$N = 15$, $x = 4$

$x^2 = 16 \equiv 1 \pmod{15}$

$(x-1)(x+1) = 3 \cdot 5 = 15 = N$

neither 3 nor 5 is divisible by 15

But

$gcd(3, 15) = 3$

$gcd(5, 15) = 5$

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Shor's quantum factoring algorithm steps

Classical processing

→ Get the number N to be factored

→ pick random integer a, $1 < a < N-1$, compute $g = \gcd(a, N)$ & If $g > 1$. you are already found a nontrivial factor.

Quantum period-Finding

→ Find out the period r such that $a^r \equiv 1 \pmod N$ using QPE

→ Arrive at the unitary matrix U ($U|y\rangle \to |ay \bmod N\rangle$)

→ Generate QPE circuit using counting registers, (QPE → Quantum phase Estimation.) work registers, apply controlled modulator exponentiation

→ Add inverse QFT module.

→ Measure the counting register (m bit outcome K & $K/2^m \approx s/r$)

Classical post processing

→ use the continued-fraction algorithm on $K/2^m$ to recover the denominator r

→ check that r is even and $a^{r/2} \not\equiv -1 \pmod N$

→ compute $\gcd(a^{r/2} \pm 1, N)$ → nontrivial factors of N

→ If any check fails, pick a new a and repeat

probability of guessing

probability of guessing the right a is $1 - \frac{1}{\ldots}$

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Grover's algorithm

→ An algorithm proposed by Lov Grover solves the problem of an unstructured search

→ it is a quantum algorithm for finding the input value x^* of a function $f(x)$ with $f(x^*) = 1$ and $f(x) = 0$ for all other values of x.

→ An example for a problem to use the algorithm is finding a phone number in an unsorted telephone directory. you need N (no. of entries) steps to get the right answer (on avg it may be N/2). with Grover's algorithm, we can get the answer with maximum of $\sqrt{N}$ steps [quadric speedup]

→ it can also be used in cryptanalysis, optimization, drug discovery etc.

Algorithm steps

→ Represents all the N input states with uniform superposition of all states (with n no of qubits and applying Hadamard gate) by 2^n $(N=2^n)$ qubit where n is an integer

$$H^{\otimes n} |0\rangle^{\otimes n} = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle = |s\rangle$$

→ Define a black box unitary U that "flip the sign" of largest states.

$$U_\omega|x\rangle = -|x\rangle \text{ for } x=\omega, \text{ that is } f(x)=1$$
$$U_\omega|x\rangle = |x\rangle \text{ for } x \neq \omega, \text{ that is } f(x)=0$$

UNIT - IV

Magnetic and Dielectric Materials

Magnetic Materials:

Introduction:

Magnetism has its own application in the field of physics. Magnetic properties have been the subject of special interest because of the information they yield about the constitution of matter. Magnetics have wide range of application in electrical machinery and in magnetic tapes in computers. Magnetic materials are classified into three, namely diamagnetic, paramagnetic and ferromagnetic materials.

Basic definitions: -

Magnetic field: The 3D region around a magnet in which it`s magnetic force is felt is called magnetic field of that magnet. It is a scalar quantity.

Magnetic dipole*:* It`s a system consisting of two equal and opposite magnetic poles separated by a small distance of $2lm$

Pole strength (m): The ability of a pole of magnet to attract or repel another magnetic pole is called it`s pole strength.

Pole strength **(m)=il**

Where i is current, l is length

Its SI unit is ampere-meter

Magnetic dipole moment (µ): It is defined as the product of pole strength and distance between them.

$$\boxed{\mu = 2lm}$$

It is a vector quantity. Its direction is from south pole to the north pole.

Its SI unit is ampere/m^2.

Magnetic flux $(\emptyset)$***:*** The total number of magnetic lines of force passing normal to a surface in a magnetic field is called magnetic flux.

Its SI unit is weber or $\text{tesla} - \text{m}^2$

Magnetic field induction or magnetic flux density (B): The magnetic flux passing through a unit normal area of substance is defined as magnetic flux density and is denoted by B. It is commonly known as magnetic induction. Let $\emptyset$ be the flux passing through a substance of area A, then flux density is

$$\boxed{B = \frac{\emptyset}{A} = \frac{F}{m} \textbf{ or } \emptyset = \mathbf{B} \cdot \mathbf{A} = BA\cos\theta.}$$

It is a vector quantity.

Its SI unit is weber/m^2 or tesla and CGS unit is Gauss ($= maxwell/cm^2$).

1Tesla= 10^4Gauss.

Magnetizing field(force) or Magnetic intensity(H): The force acting on a unit north pole placed at the point. It is independent of the medium.

$$\boxed{B = \mu H}$$

It is vector quantity. Its SI unit is ampere/m.

Permeability: It`s ability to allow the magnetic lines of force to pass through it ***or*** to allow itself to be influenced by magnetic field (μ).

Relative magnetic permeability(μ_r): The ratio of its absolute magnetic permeability(μ) of the material to the magnetic permeability of free space (μ_o).

$$\boxed{\mu_r = \frac{\mu}{\mu_0}}$$

It is purely a number; it has no units.

The permeability of free space or vacuum is $\mu_0 = 4\pi \times 10^{-7}$ H/m.

Magnetization*:* The ability of a material to get magnetized when placed in an external magnetic field is called magnetization.

It is measured by a quantity called intensity of magnetization(I).

The net magnetic dipole moment acquired by a substance per unit volume is defined as intensity of magnetization (I).

$$\boxed{\mathrm{I} = \frac{\text{net magnetic dipole moment of material}}{\text{Volume of the material}} \text{ A/m}}$$

It is vector quantity. Its SI unit is A/m.

Relation between B, H and I:

The flux density B in a material due a to magnetizing force H, as the sum of the flux density B_0 in vacuum produced by magnetizing force and flux density B_m due to magnetization of material.

Therefore $\qquad B = B_0 + B_m$

Also, we know that $B_0 = \mu_0 H$ *and* $B_m = \mu_0 \mathrm{I}$

Therefore, $\qquad B = \mu_0 H + \mu_0 \mathrm{I}$

$$B = \mu_0(H + I)$$

Magnetic Susceptibility(χ): The ratio of magnetization to the magnetic intensity.

$$\boxed{\chi = \frac{I}{H}}$$

It has no units when a material has high susceptibility then it can be easily magnetized.

Relation between Permeability and Susceptibility:

We know that $B \propto (H+I)$

$B = \mu_o (H+I)$

In vacuum $B = \mu_o H + \mu_o I$

$$\frac{B}{H} = \mu_0 + \mu_0 \frac{I}{H}$$

Also,

$\frac{B}{H} = \mu$ (permeability of the material) and $\chi = \frac{I}{H}$ (Susceptibility) $\therefore \mu = \mu_o + \mu_o\chi$ **or** $\mu = \mu_o(1 + \chi)$

$$\frac{\mu}{\mu_0} = (1 + \chi)$$

$$\mu_r = (1 + \chi)$$

$$\mu_r - 1 = \chi$$

$$\boxed{\mu_r = \chi + 1}$$

Origin of magnetic moment:

Materials are made up of atoms. These atoms consist of positively charged nucleus, surrounded by cloud of electrons. In all atoms electrons are revolving around the nucleus in different orbits and also spin about their own axis. These revolving electrons constitute an electrical current in the orbits. These revolving electrons produces its own orbital magnetic dipole moment, measured in Bohr magneton(μ_B) and there is also a spin magnetic moment associated with it (spin of orbital electrons and spin of nucleus). In most materials there is no resultant magnetic moments, due to the electrons being grouped in pairs causing the magnetic moment to be cancelled by its neighbor. Under external applied magnetic field these dipoles experience torque in the direction of applied field and the atom acquires certain magnetism.

So, the magnetic moment of an atom is due to

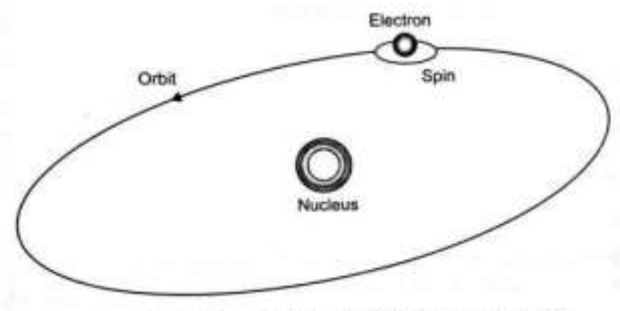
1. Orbital magnetic moment of the electrons.
2. Spin magnetic moment of the electrons.
3. Spin magnetic moment of the nucleus.

The orbital magnetic moment of the electrons(BohrMagneton):

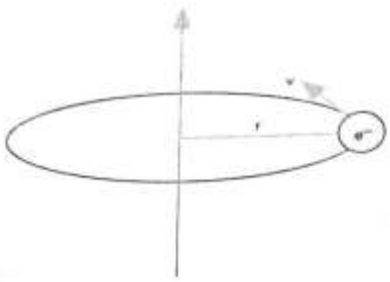
The magnetic moment contributed by a single electron is known as Bohr Magneton. Mathematically it can express as

$$\boxed{\mu_B = \frac{e\hbar}{2m}}$$

Proof:



The orbit of a spinning electron about the nucleus of an atom



Motion of electron in a orbital

Let us consider an electron revolving in a circular orbit of radius **r** with a speed $\boldsymbol{v}$. Consider a Point ***p*** on the circle. The electron crosses this point once in every revolution. In one revolution the electron travels 2πr distance.

The current $\boldsymbol{i}$ due to the electron motion in the circular loop (orbit) is

$$i = \frac{charge\ of\, electron(q)}{time(T)} = \frac{-e}{T}$$

Where, ***T*** is time for one revolution of electron about the nucleus.

$$\boldsymbol{i} = \frac{-e}{\frac{2\pi r}{v}}$$

$$T = \frac{distance\ \ travelled\ by\ an\ electron}{speed\ of\ an\ electron} = \frac{2\pi r}{v}$$

$$\frac{-ev}{2\pi r}$$

The magnetic moment μ_l associated with the orbit due to orbital motion of electron is

$$\mu_l = iA$$

Where, '$\boldsymbol{i}$' is current produced by the orbital motion of the electron and $\boldsymbol{A}$ is area covered by the orbital(πr^2).

$$\mu_l = \frac{-ev}{2\pi r} \times \pi r^2$$

Dividing and multiplying above eq. by the mass of the electron

$$\mu_l = \frac{-evr}{2} \times \frac{m}{m} = (\frac{-e}{2m})[mvr] = (\frac{-e}{2m})\,[L]$$

w.k.t the angular momentum of an electron revolving in circular orbit is equal to integral multiple of $\frac{h}{2\pi}$

$$L\ =\ mvr\ =\ \frac{lh}{2\pi}$$

$$\mu_l\ = (\frac{-eh}{4\pi m})\, l$$

$\mu_l = \mu B\ l$ where l= 0,1,2........

Where, l = orbital quantum number.

$$\mu_B = \frac{-eh}{4\pi m}$$

Bohr magneton

$$= \frac{1.6 \times 10^{-19} \times 6.626 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31}} = 9.27 \times 10^{-34} \mathrm{A} - \mathrm{m}$$

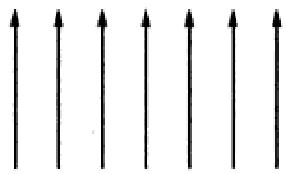
Here – sign indicates that magnetic moment is anti-parallel to the angular momentum L.

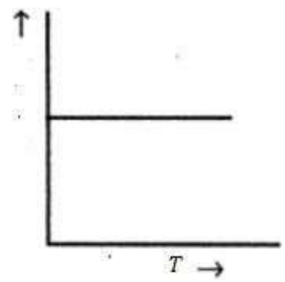
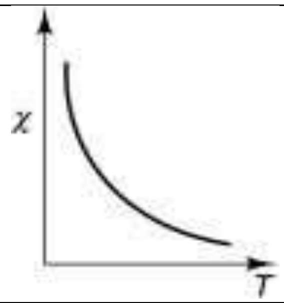
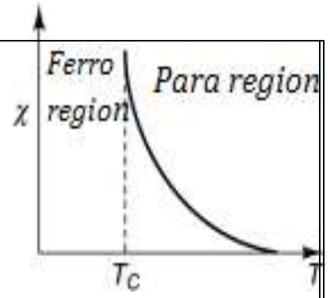
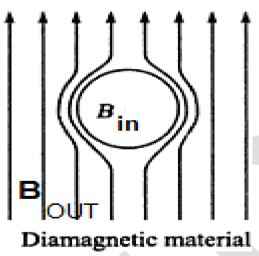
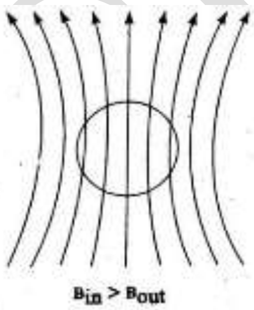
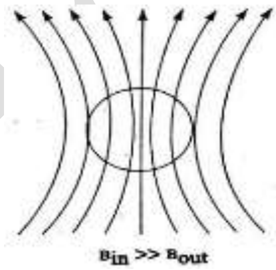
CLASSIFICATION OF MAGNETIC MATERIAL*:*

Depending on the direction and magnitude of magnetization and also the effect of temperature on magnetic properties, all magnetic materials are classified into Dia, para and ferro materials. Depending upon the spin orientation of the electrons, ferro magnetic materials are classified into two types. They are

a. Antiferromagnetic material.

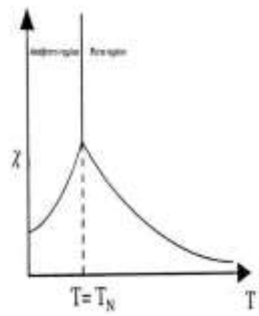
b. Ferrimagnetic materials.

PROPERTY	***DIA MAGNETIC SUBSTANCES***	***PARA MAGNETIC SUBSTANCES***	***FERRO MAGNETIC SUBSTANCES***
Definition	The substance in which the resultant magnetic moment of individual atoms is zero.	The substance in which the resultant magnetic moment of individual atoms is not zero.	The substance in which the resultant magnetic moments of individual atoms align themselves in parallel by giving rise to spontaneous magnetization
Cause	Orbital motion of electrons.	Spin motion of electrons.	Formation of domains.
***Susceptibility* (χ)**	Low and negative	Low and positive.	High and positive.
χ depend on T	Does not depend on T. except Bi at low T.	$\chi = \frac{C}{T}$ (curie law) c is curie constant.	$\chi = \frac{C}{T-T_c}$: $T > T_C$ (curie wises law) T_c is curie temperature.
***Relative permeability* (μ_r)**	<1	>1	$\gg 1$
spin alignment	No spin alignment is present.	All spins or magnetic moments are randomly oriented. 	All spins or magnetic moments are orderly oriented. 

$\chi - T$ curve			
Transition of material at T_c	Do not change.	On cooling these are converted into ferromagnetic material.	Converted into para magnetic material aboveT_c.
In the presence of magnetic field.	The magnetic lines of force are pulled out from the material .Hence, magnetic flux density is greater, outside the material than inside. 	The magnetic lines of force are attracted towards the center of the material and hence the B_{in} material is greater than the B_{out}. 	The magnetic lines of force are highly attracted towards the center of the material and hence the B_{in} material is very greater than the B_{out}. 
Example:	Bi, Zn, H_2O, Gold.....	Al, Pt, Mn , $cucl_2$....	Fe, Ni, Co, Mno, Fe_2o_3....

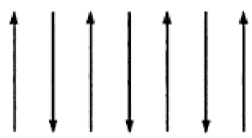
Antiferro magnetic materials:

These are the ferromagnetic materials in which the magnetic interaction between any two dipoles aligns themselves anti parallel to each other. All the dipoles are equal in magnitude. Therefore, the resultant magnetization is zero.



Properties:

1. Susceptibility is small and positive for these materials.
2. Spin alignment is systematic, but in antiparallel manner.
3. Initially, the susceptibility increases slightly as the temperature increases, and beyond a particular temperature known as Neel temperature the susceptibility decreases with temperature.
4. The temperature at which anti ferromagnetic material converts into paramagnetic material is known as Neel`s temperature.
5. Susceptibility is inversely proportional to the temperature. The variation of susceptibility with temperature is expressed as

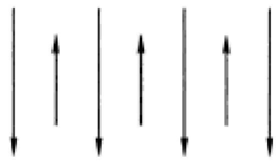


$\chi = \frac{C}{T+T_N}$ When $T > T_N$

$\chi \propto T$ When $T < T_N$

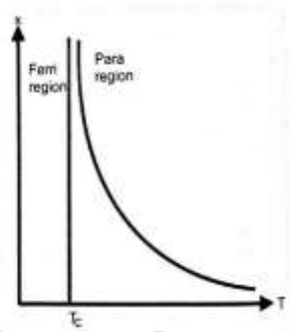
Where 'c' is the curie constant and the T_N is Neel temperature.
Ex: FeO, MnO, Cr_2O_3

Ferrimagnetic materials:



These are the ferromagnetic materials in which the magnetic interaction between any two dipoles aligns themselves anti parallel to each other. But the magnitudes of any two adjacent dipoles are not equal. Therefore, if we apply a small value of magnetic field, it will produce a large value of magnetization.

Properties:



1. Susceptibility is positive and very large for these materials.
2. Spin alignment is systematic, but in antiparallel of different magnitudes.
3. Ferrimagnetic materials possess net magnetic moment.
4. Above curie temperature becomes paramagnetic while below it behaves as ferrimagnetic material.
5. Ferrimagnetic domains become magnetic bubbles to act as memory elements.
6. Susceptibility is inversely proportional to the temperature. The variation of susceptibility with temperature is

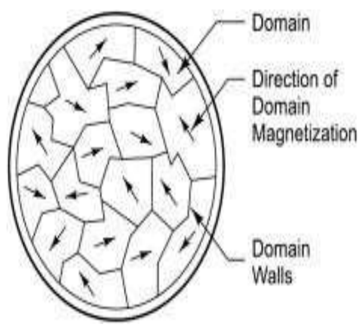
$$\chi = \frac{C}{T \pm T_N} for T > T_N$$

Where, C is curie constant and T_N is Neel temperature.

7. Ferrites are the best examples of ferromagnetic materials.
8. General formula of ferrites are $Me^{2+}OFe_2O_3$ or $Me^{2+}Fe_2O_4$

Examples: $Zn^{2+}Fe_2O_4$, $CuFe_2O_4$.......

WEISS DOMAIN THEORY OF FERROMAGNETISM:



The concept of domains was proposed by Weiss in order to explain the properties of ferromagnetic materials and their hysteresis effects.

Magnetic Domains: Every ferromagnetic material is made of a very large number of miniature (very small) regions which are known as domains. The boundaries separating the domains are called domain walls, also known as Bloch walls. In each domain the magnetic dipoles align parallel to each other and produce spontaneous magnetization. The direction of spontaneous magnetization varies from domain to domain.

Process of Domain magnetization:

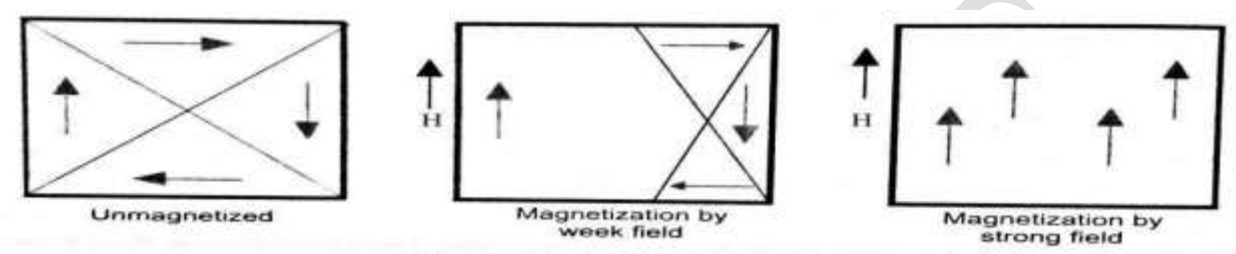
When the external field is applied there are two possible ways of alignment of random domains.

1. By the motion of domain walls.
2. By rotation of domains.

Motion of Domain walls: When a small number of magnetic fields applied on a ferro magnetic material then the domains of the material whose magnetic moments are parallel or nearly parallel to the direction of applied field can grow in size whereas the domains are not parallel to field can diminish in size. This change produces large magnetization for the bulk material.

Rotation of Domains:

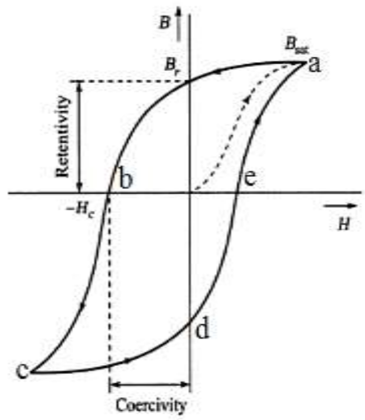
When the magnetic field is increased further to a large value (i.e. near saturation), further domains growth becomes impossible and hence fully grown domains can rotate into the field direction and specimen is said to be saturated.



Magnetization of a ferro magnetic material

HYSTERESIS CURVE:

it is defined as a curve or loop plotted on B-H co-ordinates showing how the magnetization of a ferromagnetic material varies when subjected to a periodically reversing magnetic field. This is curve used to describe a magnetic material.



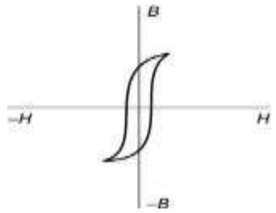
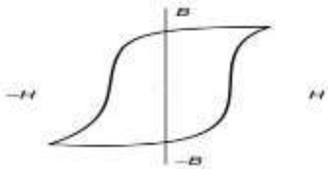
Explanation: When the magnetic field H is applied on an unmagnetized ferromagnetic material the magnetic induction increases first rapidly and then slowly from **o** to **a**. The increase is nonlinear, after that the rate of induction slows down and attaining a saturation value B_{sat}, with further increase in H, there is no increase in B. If applied magnetic field H on the material is decreased at this stage, naturally B decreases and B will not travel in that initial path, creates a new path.

When H is reduced to zero, B does not vanish, the value of B that remains in the material is called as retentivity B_r or reduce magnetization. To remove retentivity in the material sufficient negative magnetic field $-H_c$ or H_c is applied in opposite direction then the retentivity B_r becomes zero. This is (field) known as coercive field. On increasing H further B reaches saturation in the opposite direction denoted by 'c'. Then reducing H to zero B reaches 'd' and then increasing H in the positive direction B reaches again to B_{sat} and a curve is obtained. This completes a closed loop called hysteresis loop. It includes some area. This area indicates the amount of energy wasted in one cycle of operation.

SOFT AND HARD MAGNETIC MATERIALS:

Ferromagnetic materials are classified into two types based on the characteristic parameters such as hysteresis and magnetization. They are

1. Hard magnetic materials
2. Soft magnetic materials

S.NO	SOFT MAGNETIC MATERIAL	HARD MAGNETIC MATERIAL
1	Materials which can be easily magnetized and demagnetized are called Soft magnetic materials.	Materials which can't be easily magnetized and demagnetized are called hard magnetic materials.
2	The nature of hysteresis loop is very steep 	The nature of hysteresis loop is very large. 
3	They are prepared by annealing process.	They are prepared by quenching process.
4	Due to small hysteresis loop area, they have small hysteresis loss.	Due to large hysteresis loop area, they have large hysteresis loss.
5	They have large value of permeability and susceptibility.	They have low value of permeability and susceptibility.
6	The coercivity and retentivity are small.	The coercivity and retentivity are large.
7	They are free from irregularities.	They have large amount of impurities and lattice defects.
8	They are used • To produce temporary magnets. • In the preparation of magnetic core materials used in transformers, electric motors , magnetic amplifiers, magnetic switching circuits, etc.,	They are used • To produce permanent magnets. • In loud speakers, toys, in measuring meters, microphones, magnetic detectors, magnetic separators, etc.,
9	***Ex:*** Iron and silicon alloys ,Nickel – Iron alloy, Iron – Cobalt alloy.	***Ex:*** High carbon steel, Cobalt steel, Barium ferrite .

Applications of magnetic materials: -

MAGNETIC HYPERTHERMIA FOR CANCER TREATMENT

Magnetic hyperthermia is a cancer treatment technique that uses heat generated by magnetic nanoparticles to kill cancer cells.

Magnetic Hyperthermia

Hyperthermia means raising the temperature of body tissue. In magnetic hyperthermia, tiny magnetic particles (usually iron oxide nanoparticles) are introduced into the tumor. When exposed to an alternating magnetic field, these particles generate heat. The heat increases the temperature of the tumor tissue to around 41–45°C, This temperature range can damage or kill cancer cells while sparing healthy cells, which are more resistant to heat.

Working

1. Nanoparticle Injection: Magnetic nanoparticles, typically made of iron oxide (like magnetite or maghemite), are either injected directly into the tumor or delivered systemically.

2. Targeting Tumor Cells: These particles accumulate preferentially in the tumor due to enhanced permeability and retention (EPR) effect or can be functionalized to bind specifically to cancer cells.

3. Applying Alternating Magnetic Field: An external AMF is applied to the region containing the nanoparticles.
4. Heat Generation: The nanoparticles respond to the AMF by rapidly flipping their magnetic moments, producing heat via Néel and Brownian relaxation mechanisms.
5. Cancer Cell Death: The localized temperature typically rises to 41–45°C (106–115°F). This heat:
1. Damages cancer cell membranes,
2. Denatures proteins,
3. Induces apoptosis (programmed cell death),

4. Increases tumor blood flow to improve immune response or enhance other therapies

APPLICATIONS OF MAGNETS IN EV

Magnets play a crucial role in electric vehicles (EVs) in several ways, mainly related to their electric motors and energy efficiency. Some of them are listed below.

1. Electric Motors:
Permanent Magnet Synchronous Motors (PMSMs): These motors use strong permanent magnets (often rare-earth magnets like neodymium-iron-boron, NdFeB) in the rotor. The magnets create a constant magnetic field, which interacts with the stator's rotating magnetic field to produce torque.
Brushless DC Motors (BLDC): Similar to PMSMs, these motors use permanent magnets on the rotor for efficient and compact design.
2. Generators:
In regenerative braking systems, magnets in generators convert kinetic energy back into electrical energy, improving overall energy efficiency.
3. Sensors:
Magnetic sensors (like Hall effect sensors) monitor rotor position and speed to control motor operation precisely. Used in battery management systems for current and voltage sensing.
4. Magnetic Gears and Couplings:
Magnetic gears use magnetic fields to transfer torque without direct mechanical contact, reducing wear and noise.
Can improve efficiency and reliability in drivetrain systems.

5. Wireless Charging:
Magnetic induction coils are used in wireless charging pads and receivers for EV batteries, allowing convenient and cable-free charging.
Magnets are important in EV Motors because they are used to:
1. Provide high torque density with less weight.
2. Enable compact and lightweight motor designs.
3. Improve energy efficiency, extending EV driving range.
4. Offer precise control of motor speed and torque.

GIANT MAGNETO-RESISTANCE (GMR) DEVICE

Giant Magnetoresistance (GMR) devices exploit the large change in electrical resistance caused by the alignment of magnetic layers separated by a thin non-magnetic layer. This phenomenon is crucial in various high-tech applications, especially where sensitive magnetic detection is needed.

Applications of GMR devices

1. Magnetic Read Heads for Hard Disk Drives (HDDs)
- GMR sensors are used to read data from the magnetic storage media.
- The change in resistance in the GMR sensor corresponds to the magnetic field from the disk surface, allowing high-density data storage and fast data retrieval.
- Revolutionized HDD technology by enabling much higher storage capacities.

2. Magnetic Random Access Memory (MRAM)
- GMR effects are used in MRAM to read data.

- MRAM stores data in magnetic elements, offering non-volatile memory with fast read/write speeds and high endurance.
- Potential future replacement for traditional RAM and flash memory.

3. Magnetic Field Sensors
- GMR sensors can detect small magnetic fields with high sensitivity.
- Used in automotive applications (e.g., wheel speed sensors, crankshaft and camshaft position

sensors).
- Industrial applications include current sensing and proximity sensing.

4. Bio-Magnetic Sensing
- GMR devices are used in biosensors for detecting magnetic nanoparticles tagged to biological molecules.
- Applications in medical diagnostics, such as detecting specific proteins or DNA sequences.

DIELECTRIC MATERIALS

1.1 Introduction

- Dielectrics are insulating materials. In dielectrics, all electrons are bound to their parent molecules and there are no free charges.
- Even with normal voltage or thermal energy electrons are not released.
- Dielectrics are nonmetallic materials of high specific resistance and have negative temperature coefficient of resistance.
- Dielectrics are electrical insulators. They possess high resistivity values within the range 10^6 Ωm to 10^{16} Ωm. Under high voltage bias, they allow very little current. They with stand for very high voltages. The conduction is mostly associated with ionic motion through defects or hopping of charges. They have no free charges. The electrical properties of a dielectric are associated with inherent property of possessing electric dipoles.
- Dielectrics are the materials having electric dipole moment permanently or temporarily by applying electric field. These are mainly used to store electrical energy and as electrical insulators. All dielectrics are electrical insulators. But all electrical insulators need not be dielectrics. For example, the vacuum is a perfect insulator. But it is not a dielectric. The study of dielectrics is essentially study of insulators.

1,2 Basic Definitions

Electric dipole

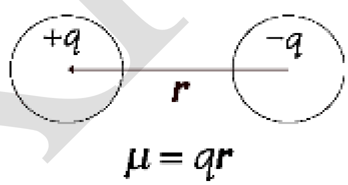


Fig. 1.1 Electric dipole

Two equal and opposite charges separated by a distance ‘r’ constitute a dipole.

Electric dipole moment (μ)

The product of charge and distance between two charges is called electric dipole moment.

$\mu = q \times r$

Units: coulomb - meter or Debye.

1 Debye = 3.33×10^{-30} coulomb - meter

Non-polar dielectrics

Mono atomic materials are made up of atoms. The centre of gravity of negative charge and the centre of gravity of positive charge of an atom coincide. That means even though there are two equal and opposite charges are not separated. Their dipole moment is zero.

$\therefore\ \mu = q \times r = q \times 0 = 0$

Such dielectrics are called Non- polar dielectrics.

Non-Polar Molecule

- Consider an atom. The positive charge of nucleus may be concentrated at a single point called as centre of gravity of the positive charge.
- The negative charge of electrons may be supposed to be concentrated at a single point called as Center of gravity of the positive charge.
- When the two centre of gravity coincide, the molecule is known as Non-polar molecule. The non-polar molecules have symmetrical structure and zero electric dipole moment.
- Examples: H_2, N_2, O_2, CO_2, Benzene.

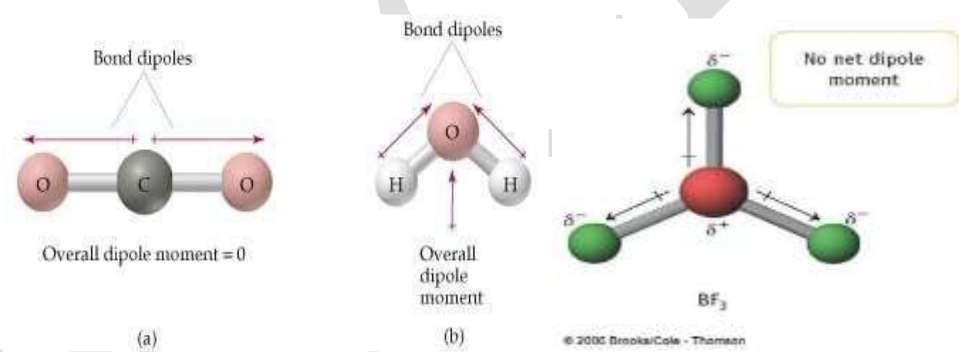


*Fig. 1.2 Non polar molecules with zero dipole moment **(μ=0)***

- The electric dipole moment has a direction from positive charge to negative charge.

Polar dielectrics

- In polyatomic molecules, the center of gravity of negative charge distribution may not coincide with the center of positive charge distributions.
- There is an effective separation between centers of negative and positive charge distributions.
- The molecule has a net dipole moment. Such dielectrics are called Polar dielectrics.

Polar Molecules

They have unsymmetrical structure and have a permanent electric dipole moment. The Center of gravity of positive and negative charges do not coincide, the molecule is called as polar molecule.

e.g.:- H_2O, HCl, CO, N_2, NH_3 etc.

*Fig.1.3 Polar molecules with net dipole moment (**μ≠0**)*

Dielectric constant $\in_r$ (or) Relative permittivity of the medium

- It is the ratio between the permittivity of the medium and the permittivity of free space.

$\epsilon_r = \frac{\epsilon}{\epsilon_0}$. It has no units.

- Єr is also called as relative permittivity of the medium. It is a measure of polarization in the dielectric material.

$\epsilon_r = \epsilon_0 \epsilon_r$ where Є = absolute permittivity of the medium
Where Єo = permittivity of free space=8.854 x 10^{-12} F/m

Electric Polarization

When a dielectric substance is placed in an electric field, then positive and negative charges are displaced in opposite direction.

- The displacement of charges produces local dipoles.
- This process of producing dipoles by the influence of electric field is called electric polarization.

$$\text{dielectric Polarization} = P = \frac{electric\ dipolemoment}{volume}$$

$$P = \frac{\mu}{V}$$

$$P = \frac{q \times l}{A \times l} = \frac{q}{A}$$

$$P = \frac{\text{charge}}{\text{area}} = \text{surface charge density '}\sigma\text{'}$$

Polarizability (α)

The average dipole moment $\bar{\mu}$ is directly proportional to the electric field (E) applied.

$$\mu \; \alpha \; E$$

$$\mu = \propto E$$

$\propto$ = Polarizability = $\frac{\mu}{\vec{E}}$ Farad/m^2

Polarization Vector (P)

It is defined as the average dipole moment per unit volume of a dielectric. If 'N' molecules are present per unit volume,

Then polarization vector $\vec{P} = \frac{N\bar{\mu}}{volume}$

$$\vec{P} = N\bar{\mu} \quad \text{coulomb/m}^2$$

Electric flux density (or) Electric displacement (D)

The number of electric lines of forces received by unit area is called Electric flux density.

$$D \; \alpha \; E$$

$$D = \epsilon E$$

$$\text{But } \epsilon = \epsilon_0 \epsilon_r$$

$$D = \epsilon_0 \epsilon_r E$$

Relation between Polarization (P), Electric field (E) & Dielectric constant ϵ_r

We know that electric flux density 'D' is written as

$$D = \epsilon E \qquad \rightarrow (1)$$

Where ϵ= absolute permittivity of medium

E = electric field

Where $E = \epsilon_0 \epsilon_r \qquad \rightarrow (2)$

Hence substitute (2) in (1)

$$D = \epsilon_0 \epsilon_r \qquad \rightarrow (3)$$

Where ϵ_0 = permittivity of free space = 8.854 x10^{-12} F/m

ϵ_r = Dielectric constant or relative permittivity of the medium.

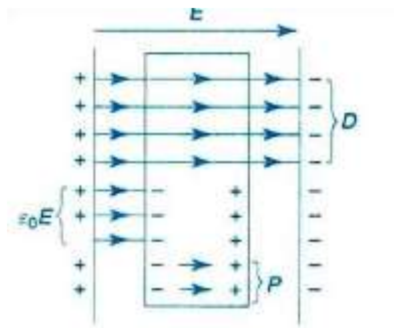

E
D
$\varepsilon_0 E$
P

Fig. 1.4 Electric lines of forces in a polar dielectric

If ‘P’ is the polarization of the dielectric material due to the applied electric field (E), then the flux density ‘D’ is equal to the flux density in vacuum plus polarization of the material

$\therefore D = \epsilon_0 E + P$ → (4)

equating equations (3)& (4)

$$\epsilon_0 \epsilon_r E = \epsilon_0 E + P$$

$$P = \epsilon_0 E [\epsilon_r - 1]$$

$P = \epsilon_0 \epsilon_r E - \epsilon_0 E$ → (5)

Put $[\epsilon_r - 1] = \chi$ → (6)

Where χ = electric susceptibility

from equation (5) & (6)

$$\Longrightarrow P = \epsilon_0 E \chi$$

$\chi = \frac{P}{\epsilon_0 E}$ → (7)

Conclusion:

$$\Rightarrow P = E [\epsilon_r - 1]$$

$$\chi = \epsilon_r - 1$$

$$\chi = \frac{P}{\epsilon_0 E}$$

Electric Susceptibility (χ)

The electric susceptibility 'χ' is defined as the ratio of polarization vector to the applied electric field 'E'.

$$\chi = P/\epsilon_0 E$$

χ has no units.

$$\Rightarrow P = \chi\, \epsilon_0 E$$

$$\text{Where } \chi = \epsilon_r - 1$$

1.3 Non-Polar Dielectric in an electric field

When a dielectric is placed in an electric field, say between the plates of a charged Condenser; the positive and negative charges are re oriented i.e. the center of gravity of positive charges is pulled towards the negative plate of the condenser and vice versa. Thus the net effect of the applied field is to separate the positive charges from the negative charges. This is known as Polarization of dielectric. The dielectrics which are polarized only when they are placed in an Electric field are called Non-polar dielectrics.

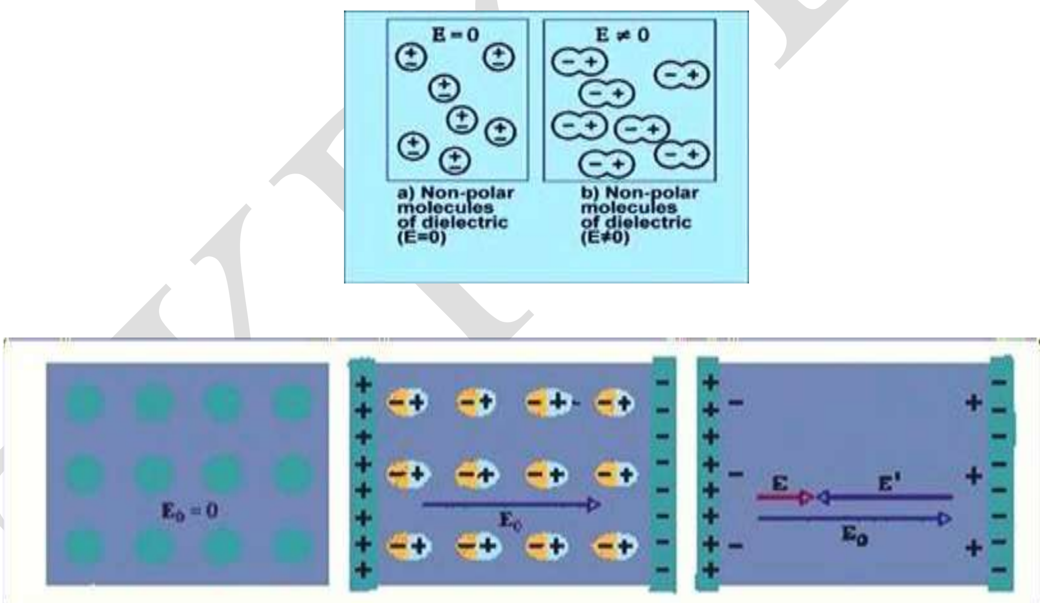


Fig. 1.5 (a) non polar dielectric in E=0 (b) Non polar dielectric when 'E' is applied (c) Totally polarized non polar dielectric with net field

Thus if the dielectric is placed in an electric field, induced surface charges appear which tend to weaken the original field with in the dielectric. That means E^1 opposes the original field E_o.

1.4 Polar dielectric in electric field

We know that polar dielectric have permanent dipole moments with their random orientations. In the presence of an electric field, the partial alignment of dipoles takes place.

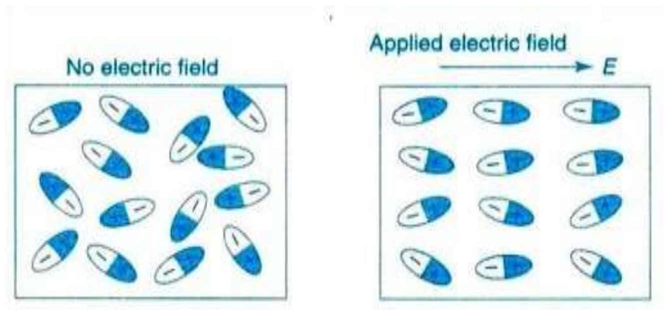


Fig. 1.6 Polar dielectric orientation without field and with field.

Polar dielectrics already possess some dipole moment inside due to the presence of permanent atomic dipoles. But these are randomly oriented when no field is applied. Their dipole moment and polarization increases since dipoles align along the field direction gives some extra polarization. Hence

$$P = \left\{ \begin{array}{l} \text{Polarization already} \\ \text{Existing due to permanent} \\ \text{dipoles} \end{array} \right\} + \left\{ \begin{array}{l} \text{Polarization induced} \\ \text{due to electric} \\ \text{field} \end{array} \right\}$$

$$P = P_p + P_i$$

1.7 Types of Polarization

Polarization is the process of inducing dipole moment in a molecule. There are four types of polarization. They are:

(1) Electronic Polarization
(2) Ionic Polarization
(3) Orientation (or) Dipolar Polarization
(4) Space charge polarization

1.7.1 Electronic Polarization:

Definition:

When an electric field is applied on a dielectric material then all the positive nuclei of atoms move in the field direction and all the negative electron cloud of atoms move in opposite directions, hence dipoles are formed to produce dipole moment.

- The electron cloud readily shifts towards the positive end of the field. The extent of shift by electrons is proportional to field strength.
- Hence dipole moment is the product of charge and shift distance.

Expression for Electronic Polarizability α_e

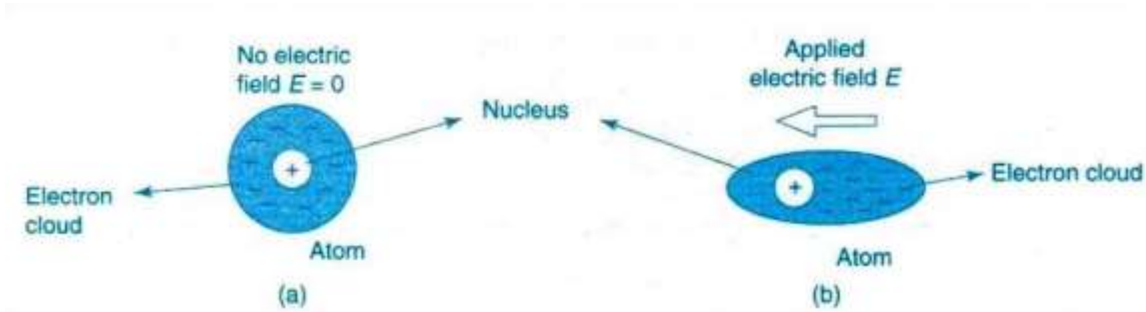


Fig. 1.8 (a) Un polarized atom in the absence of field (b)Electronic polarization due to distortion of electron cloud by the field E

- Consider an atom of dielectric material such that its atomic radius is ‘R’ & atomic number ‘Z’. When no field is applied the charge centers of electron cloud and positive nucleus are at the same point & hence there is no dipole moment.
- Suppose an atom d is placed in an electric field of strength E, due to Lorentz force the positive nucleus move towards the field direction and electron cloud will move in opposite direction of the field.
- When nucleus and electron cloud are displaced from their equilibrium positions, an attractive Coulomb force act between them to maintain original position. But Lorentz force will tend to separate them from them equilibrium position.
- When these forces are equal and opposite produces a net dipole moment.

Let the displacement between centers of nucleus and electron could be ‘X’.
Let ‘ze’ is charge of nucleus.
‘-ze’ is charge of electron cloud in a sphere of radius ‘R’.
⟹ The charge density due to negative electron cloud of radius R (E=0) is

$$\rho = \frac{(-ze)}{(\frac{4}{3}\pi r^{3})}$$

$$\rho = \frac{-3\ ze}{4\ \pi R^3} \quad \rightarrow (1)$$

Lorentz force which tends to separate positive nucleus from negative electron cloud is

$$= (-ze)\,E \quad \rightarrow (2)$$

Coulomb force which tends force of attraction between them is

$$\Longrightarrow (ze)\frac{[\ charged\ enclosed\ in\ sphere\ of\ radius\ x]}{4\pi\epsilon o x^2} \quad \rightarrow (3)$$

But the charge enclosed in the sphere of radius ‘x’ due to electrons of radius ‘x’

$$= \frac{4}{3}\pi x^3 \rho$$

$$= \frac{4}{3}\pi x^3 \left[\frac{-3}{4}\ \frac{ze}{\pi R^3}\right] \{from\ equation\ (1)\}$$

$$= \frac{-zex^3}{R^3} \quad \rightarrow (4)$$

Substitute equation (4) in (3) we get

$$(ze)\left[\frac{-zex^3}{3}\right]$$

$$\text{Coulomb force} = \frac{R}{4\pi\epsilon_0 x^2}$$

$$\text{Coulomb force} = \frac{-z^2 e^2 x}{4\pi\epsilon_0 R^3}$$

In equilibrium position coulomb & Lorentz forces are equal.

∴ Comparing equations (5) & (2) we get

$$\frac{x}{4\pi\epsilon_0 R^3} = (-ze)\,E \qquad \rightarrow (5)$$

Hence electron cloud displacement 'X' = $\left[\frac{4\pi\epsilon_0 R^3}{ze}\right]E$ $\rightarrow (6)$

The two charges (+ze) of nucleus & (-ze) of electron cloud are displaced through 'X'. Hence electric dipole moment is $\mu_e = |\text{charge}| \times (\text{displacement})$

$$= |ze|\,(X)$$

$$= (ze)\left(\frac{4\pi\epsilon_0 R^3}{ze}\right)E \qquad [i.e\ from\ equation\ (6)]$$

$$\mu_e = (4\pi\epsilon_0 R^3)\ E \qquad \rightarrow (7)$$

We know that electronic polarizability

$$\alpha_e = \frac{dipole\ electric\ moment}{electric\ field}$$

$$\alpha_e = \frac{(4\pi R^3)E}{E} \qquad [i.e\ from\ equation\ (7)]$$

$\Rightarrow$ Electronic Polarisability $\alpha_e = 4\pi\epsilon_0 R^3$

Where R= radius of an atom.

1.7.2 Ionic Polarization

Ionic polarization takes place in ionic dielectrics due to displacement of positive and negative ions by the influence of external electric field.

Expression for Ionic Polarisability

When an electron field is applied on an ionic dielectric then positive ions move in the field direction & negative ions move in opposite direction, hence dipoles will be formed. This phenomenon is known as ionic polarization.

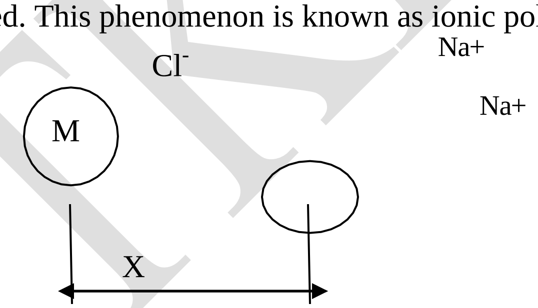


Fig. 1.9 (a) In the absence of field

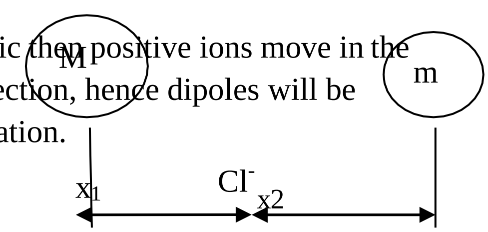


Fig. 5.9(b) when field is applied

Let 'e' the charge of ions and M and m be the masses of negative and positive ions respectively. When an electric field 'E' is applied on an ionic dielectric then positive ions displaces in the field direction through x_1 distance and negative ion displaces in opposite direction through x_2 distance.

The induced dipole moment

$$\mu = |\text{charge}| \times \text{displacement}$$

$$\mu = |e|\,(x_1 + x_2) \qquad \rightarrow (1)$$

But against to the displacement of positive and negative ions, restoring force acts and opposes the displacements of cation and anion.

Under equilibrium conditions, the restoring force

$$F = k_1\, x_1 \quad \text{[for negative ion]}$$
$$F = k_2\, x_2 \quad \text{[for positive ion]}$$

$$\left.\begin{aligned} &\Rightarrow x_1 = \frac{F}{k_1} \\ &x_2 = \frac{F}{k_2} \end{aligned}\right\} \rightarrow (2)$$

where k_1 k_2 are force constants

But

$$\left.\begin{aligned} k_1 &= M\omega_0^2 \\ k_2 &= m\omega_0^2 \end{aligned}\right\} \rightarrow (3)$$

where ω_0= angular velocity of the ions

Substituting (3) in (2) we get

Hence $x_1 = \frac{F}{M\omega_0^2}$ & $x_2 = \frac{F}{m\omega_0^2}$

But F = e E

$$\left.\begin{aligned} &\Rightarrow x_1 = \frac{eE}{M\omega_0^2} \\ &x_2 = \frac{eE}{m\omega_0^2} \end{aligned}\right\} \rightarrow (4)$$

Substituting equation (4) in equation (1)

$\Rightarrow$ Electric dipole moment

$$\mu = e[x_1 + x_2]$$

$$\mu = e\left[\frac{eE}{M\omega_0^2} + \frac{eE}{m\omega_0^2}\right]$$

$$\mu = \frac{e^2 E}{m\omega_0^2}\left[\frac{1}{M} + \frac{1}{m}\right] \rightarrow (5)$$

We know that the Ionic Polarizability

$$\alpha_i = \frac{dipole\ moment}{electric\ field} = \frac{\mu}{E}$$

$$\alpha_i = \frac{\frac{e^2 E}{\omega_0^2}\left[\frac{1}{M} + \frac{1}{m}\right]}{E}$$

$$\alpha_i = \frac{e^2}{\omega_0^2}\left[\frac{1}{M} + \frac{1}{m}\right]$$

Here ω_0 = natural frequency of the ionic molecule.

1.7.3 Orientational Polarization

Definition:

When Electric field is applied on a polar dielectric then all the dipoles tend to rotate In the field direction, hence dipole moment increases gradually. This phenomenon is known as dipolar (or) orientational polarization.

Expression for Orientational (or) dipolar Polarizability

Orientation Polarization takes place only in polar dielectrics in which dipoles orient in random manner such that the net dipole moment is zero. When Electric field is applied, all the dipoles try to rotate in the field direction as shown in the figure 5.10.

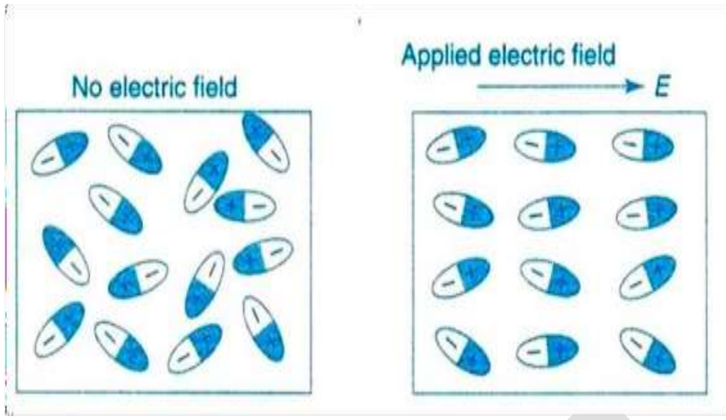


Fig. 1.10: Orientational polarization

To derive an expression for dipolar Polarisability of a system, we have to know the total dipole moment and total number of dipoles present in the system since

$$\text{Average dipole moment } \mu = \frac{Total\ \ dipole\ \ moment}{Total\ number\ of\ dipoles}$$

$$\alpha_0 E = \frac{\mu^2}{3K_B T} E$$

$$\alpha_d = \alpha_0 = \frac{\mu^2}{3K_B T}$$

α_0= Orientational (or) dipolar polarisability

μ= dipole moment

K_B= Boltzmann constant

T= absolute temperature.

Ferroelectric material: -

Ferroelectric materials can be defined as dielectric materials in which polarization remains permanently even after removing the applied electric field. Moreover, the direction of the dipole moment can be switched by applying field.

They may display permanent electric behaviour. Ferroelectric materials possess some important characteristics, which are as follows:

(i) Ferroelectric materials have permanent electric dipoles; hence polarisation occurs under the action of internal processes of the material without the influence of any external electric field.

(ii) In such materials, the relation between polarisation P and applied field Eis usually non-linear (complex) in nature.

(iii) Usually, the dielectric constants of ordinary dielectrics do not vary much with temperature. But the dielectric constants of ferroelectric materials depend highly on temperature.

(iv) The dielectric constants of ferroelectrics drop significantly above a critical temperature known as Curie temperature, which is also known as transition temperature.

(v) Ferroelectric materials show the electric hysteresis for the variation of polarisation with the applied field. This property of ferroelectric materials is analogous to the ferromagnetic materials which exhibit magnetic hysteresis for the variation of B versus H.

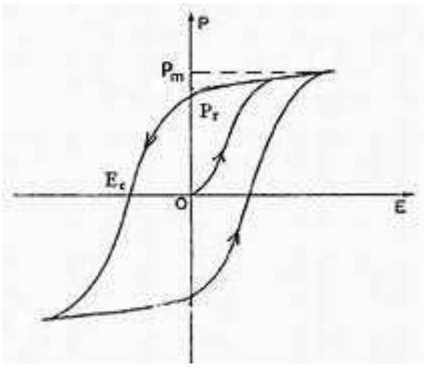
For many years, the phenomena of ferroelectricity were known to occur in a rather limited number of crystals, viz., Rochelle salt, potassium dihydrogen phosphate, barium titanate and a number of compounds, isomorphic with these crystals. At this stage, various classifications of the ferroelectric materials were proposed in order to facilitate the treatment of their properties.

In ferroelectric materials, there must be permanent dipoles, the origin of which can be explained with the help of barium titanate (BaTiO3), one of the most common ferroelectrics

Ferroelectric materials are that class of dielectric material which possess electrical properties which are analogous to the magnetic properties of a ferromagnet material.

Hysteresis in Ferroelectrics

Most of the dielectric materials polarise linearly with the external field, but the ferroelectric materials have non-linear dependence of polarisation on the electric field. The relationship between P and E is not only non-linear but also shows hysteresis effect, i.e., the polarisation produced by the given electric field also depends on the past history of the material. When a ferroelectric material is subjected to electric field, then the graph that is traced between polarisation P and electric field E describes a closed loop known as hysteresis loop as shown in Fig.



Hysteresis loop between polarisation and electric field

We will now discuss the loop traced between $\vec{P}$ and $\vec{E}$. Initially, the specimen has polarisation equal to zero. Under the influence of applied field, the polarisation increases along the curve OABC. After point C, polarisation becomes almost constant, i.e., it does not increase with increase in electric field.

Examples:

1. Barium titanate [BaTiO3]

2. Potassium dihydrogen phosphate [KH2PO4]

3. Rochelle Salt (NaKC4H4O6.4H2O).

Applications: -

1. Due to high value of dielectric constant, they are used to manufacture small sized capacitor of large capacitances.

2. using the hysteresis curve, they can be used as memory devices in computers.

3. they are used to detect infrared radiation

4. they are used in electromechanical fellers.

Piezoelectric Materials

Piezo is a Greek word for pressure. Thus, piezoelectricity means electricity from pressure. Curie brothers discovered the phenomenon of piezoelectricity in 1880.

These materials have the property of becoming electrically polarized when mechanical stress is applied - this property is known as piezoelectric effect and has an inverse. According to inverse piezoelectric effect when an electric stress (voltage) is applied, the material becomes strained. The strain is directly proportional to the applied field E.

When piezoelectric crystals are subjected to compression or tension, opposite kind of charges are developed at the opposite faces perpendicular to the direction of the applied force. The charges produced are proportional to the applied force. This property is utilized in conversion of mechanical energy into electrical energy and vice versa.

The quartz crystal is a very good example of one such transducer material. Crystal faces are to be cut in proper direction and shaped to achieve maximum efficiency.

Induced polarisation and the electric charge appearing on the surface of dielectrics due to mechanical strain are responsible for piezoelectricity. It has been observed that all ferroelectric materials are piezoelectric but the converse is not true.

In order to understand the effects of piezoelectricity in quartz crystal, it is necessary to have a clear idea of the geometry of the crystal and the method in which crystal plates are cut from the natural crystals.

Examples:

1. Quartz crystal

2. Rochelle salt

3. Barium titanate

4. Lead zirconate

5. Lead titanate

6. GaAs, ZnO, CdS

Applications of Piezoelectricity

Piezoelectricity has many applications. Some important applications are as follows:

1. It is frequently used in transducers, sensors, motors, clocks, and lighters.
2. It is used in medical ultrasound devices.
3. Piezoelectricity is used to convert mechanical pulses into electrical pulses and vice versa.
4. It is used in SONAR applications.
5. It is used in loudspeakers to convert electricity into sound.
6. Some piezoelectric materials can produce very high voltage. The spark produced due to high voltage is used to ignite gas fire and gas ovens.
7. Rochelle salt is used as transducer in gramophone pick-ups, ear phones, hearing aids, microphones.
8. barium titanate, lead zirconate and lead titanate are used for high voltage generation, accelerometers, transducers.
9. electric semiconductor used in amplifiers & ultrasonic wave.

Pyroelectric materials: -

The process of change of spontaneous polarization in the specimen with temperature is known as pyroelectric effect. Pyroelectricity can be described as the ability of certain materials to generate a temporary electric field when they are heated or cooled. The change in temperature modifies the positions of the atoms slightly within the crystal structure so that the polarization of the material changes. This polarization change gives rise to a voltage across the crystal. Note that pyroelectricity is different from thermoelectricity. The pyroelectric coefficient λ is defined as the rate of change of polarization vector of the material with respect to temperature.

$$\lambda = dp / dT$$

change in polarization results in change in external field and hence charge on the surface. As
q = ks

possible to detect a charge of 10^{-16} coulomb with a suitable electrode, temperature changes a small as 10^{-6} °C can be measured using the pyroelectric effect.

Some of the important pyroelectric materials are-

1. Tourmaline

2. Gallium nitride

3. Cesium nitrate ($CsNO_3$)

4. Polyvinyl fluorides

5. Derivatives of phenyl pyridine

6. Cobalt phthalocyanine

7. Lithium tantalite ($LiTaO_3$)

Applications of Pyroelectricity

Pyroelectric materials have a variety of applications. Some of them are listed below.

1. PIR-Based Motion Detectors: Pyroelectric sensors are commonly used in passive infrared (PIR) motion detectors. These detectors sense changes in infrared radiation caused by moving objects, making them useful for security systems and automatic lighting control.
2. Radiometry: Pyroelectric materials play a role in radiometry, which involves measuring electromagnetic radiation. They can be used in devices that detect and quantify radiation levels, such as infrared cameras and spectrometers.
3. Solar Energy Pyroelectric Converters: Researchers are exploring pyroelectric materials for converting heat energy (such as sunlight) directly into electricity. These materials could enhance harvest of solar energy harvesting and improve the efficiency of solar panels.
4. Detection and Protection of Wildlife: Pyroelectric sensors are employed in wildlife monitoring systems. They can detect the presence of animals based on their body heat, helping with conservation efforts and preventing human–wildlife conflicts.
5. PIR Remote-Based Thermometers: Pyroelectric sensors are used in non-contact thermometers. These thermometers measure temperature by detecting infrared radiation emitted by the object being measured.
6. Fire Detectors: Pyroelectric sensors can be part of fire detection systems. They respond to sudden temperature changes caused by flames or smoke, triggering alarms and alerting occupants.
7. Laser Diagnostics: Pyroelectric materials are used in laser diagnostic equipment. They can detect laser pulses and measure their energy, aiding in laser research and medical applications.

8. Pyroelectric materials have been used to generate large electric fields necessary to steer deuterium ions in a nuclear fusion process. This is known as pyroelectric fusion.

Applications dielectric materials

FERROELECTRIC RANDOM-ACCESS MEMORY (FE-RAM)

Fe-RAM (or FRAM) is a type of non-volatile memory that uses a ferroelectric material as its capa-

to store data. Unlike conventional RAM, Fe-RAM retains data even when power is turned off, similar to flash memory, but with distinct advantages.

Fe-RAM stores data by switching the polarization state of a ferroelectric material (often lead zirconate titanate, PZT). The two polarization states represent binary “0” and “1”.

When an electric field is applied, the polarization direction switches, allowing data to be written or read.

Key Features:

- Non-volatile: retains data without power

- Fast read/write speeds, close to DRAM

- Low power consumption during write cycles

- High endurance (can endure billions of write cycles)

- Radiation resistant, useful in harsh environments

Applications

1. Embedded Systems

Used in microcontrollers for data logging and parameter storage where low power and high endurance are important.

2. Smart Cards and RFID Tags

Reliable non-volatile memory with fast access and long life.

3. Automotive Electronics

Suitable for applications needing robust, low-power memory such as engine control units (ECUs), airbag systems, and advanced driver-assistance systems (ADAS).

4. Industrial Control Systems

Used in systems requiring frequent data writes without data loss during power outages.

5. Medical Devices

Implantable devices and portable medical equipment benefit from Fe-RAM's reliability and low power needs.

6. Consumer Electronics

Smart meters, digital cameras, and portable devices that need durable, energy-efficient memory.

7. Space and Military Applications

High radiation tolerance makes Fe-RAM ideal for aerospace and defense systems.

LOAD CELL

A load cell is a sensor or transducer that converts a force (load or weight) into an electrical signal. This signal can then be measured and displayed using an electronic system.

Load cells are typically based on strain gauge technology, where the strain (deformation) in a metal element under load changes the electrical resistance of attached strain gauges. This resistance change is proportional to the force applied.

Types of Load Cell

1. Strain Gauge Load Cells
2. Hydraulic Load Cells
3. Pneumatic Load Cells
4. Capacitive and Piezoelectric Load Cells

Applications of Load Cells

1. Weighing Systems
   - Industrial Scales: For tanks, hoppers, silos, and platforms.
   - Retail Weighing Machines: In supermarkets and shops.
   - Vehicle Weighing: Weighbridges, axle load measurement.
2. Manufacturing and Automation
   - Process Control: Monitoring force in pressing, stamping, and cutting operations.
   - Robotics: Measuring applied force during handling or assembly.
   - Quality Control: Ensuring proper assembly torque or clamping force.
3. Medical Devices
   - Patient Lifts: Monitoring patient weight.
   - Prosthetics and Orthotics: Measuring force exerted during movement or walking.
4. Aerospace and Automotive Testing
   - Crash Testing: Measuring impact forces.
   - Component Testing: Stress and fatigue analysis of mechanical parts.
5. Structural Monitoring
   - Bridge Load Monitoring: Detecting stress and strain over time.

- Building Safety: Monitoring loads in beams and columns.

6. Agriculture

- Load Monitoring in Tractors and Harvesters.

- Livestock Weighing Systems.

7. Sports Science and Biomechanics

- Measuring force output in athletic training or rehabilitation.

FIRE SENSOR

A dielectric fire sensor is a type of fire detection device that uses changes in the dielectric properties of materials (usually air or smoke particles) to detect the presence of fire or smoke. When smoke or combustion particles enter the sensor's detection chamber, they change the dielectric constant of the air, which alters the sensor's electrical characteristics. The sensor detects this change and triggers an alarm, indicating the presence of smoke or fire.

Fire sensors use different physical principles depending on the type of sensor, such as detecting:

- Smoke particles (smoke detectors)

- Heat or temperature rise (heat detectors)

- Flames or infrared/ultraviolet light emitted by fire (flame detectors)

- Gas products released during combustion (gas sensors).

Types of Fire Sensors

1. Smoke Detectors:

- Ionization smoke detectors

- Photoelectric (optical) smoke detectors

- Laser smoke detectors

2. Heat Detectors:

- Fixed temperature detectors

- Rate-of-rise detectors

3. Flame Detectors:

- Ultraviolet (UV) flame sensors

- Infrared (IR) flame sensors

- UV/IR combined sensors

4. Gas Detectors:

- Detect gases like carbon monoxide (CO), carbon dioxide (CO_2), or other combustion products.

Applications of Fire Sensors

1. Residential Buildings:

   - Early fire detection systems in homes and apartments.

2. Commercial and Office Buildings:

   - Fire alarm systems integrated with building safety protocols.

3. Industrial Facilities:

   - Monitoring hazardous processes, warehouses, and manufacturing.

   - Preventing fires in chemical plants, refineries, and factories.

4. Transportation:

   - Fire detection in airplanes, trains, ships, and vehicles.

5. Public Spaces:

   - Airports, shopping malls, schools, hospitals, and theaters.

6. Data Centers and Server Rooms:

   - Sensitive detection to protect expensive and critical IT equipment.

7. Forestry and Outdoor Areas:

   - Early wildfire detection systems.

8. Military and Aerospace:

   - Fire detection in sensitive environments with explosive or flammable materials.

UNIT V LASERS

INTRODUCTION:

LASER stands for Light Amplification by Stimulated Emission of Radiation. Laser technology started with Albert Einstein in 1917, he has given theoretical basis for the development of Laser. The technology further evolved in 1960 when the very first laser called Ruby Laser was built at Hughes Research Laboratories by T.H. Mainmann.

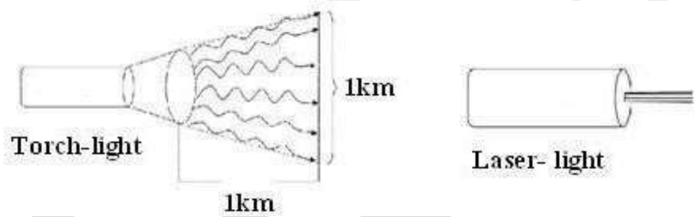
CHARACTERISTIC OF LASER RADIATION:

The laser beam has the properties given below which distinguish it from an ordinary beam of light. Those are

1. Highly directional
2. Highly monochromatic
3. Highly intense
4. Highly coherence

1. Highly directional:

A conventional light source emits light in all directions. On the other hand, Laser emits light only in one direction. The width of Laser beam is extremely narrow and hence a laser beam can travel to long distances without spreading.



The directionality of laser beam is expressed in terms of divergence

$$\Delta\theta = \frac{r_2 - r_1}{d_2 - d_1}$$

Where r_1 and r_2 are the radii of laser beam spots at distances of d_1 and d_2 respectively from laser source.

2. Highly monochromatic:

A monochromatic source is a single frequency or single wavelength source of light. The laser light is more monochromatic than that of a convectional light source. This may be due to the stimulated characteristic of laser light. The band width of convectional monochromatic light source is 1000A^0. But the band width of ordinary light source is 10 A^0. For high sensitive laser source is 10^{-8} A^0.

3. Highly intense:

Laser light is highly intense than the conventional light. A one milli-Watt He-Ne laser is highly intense than the sun intensity. This is because of coherence and directionality of laser.

Suppose when two photons each of amplitude 'A' are in phase with other, then young's principle of superposition, the resultant amplitude of two photons is 2A and the intensity is $4a^2$. Since in laser many numbers of photons are in phase with each other, the amplitude of the resulting wave becomes 'nA' and hence the intensity of laser is proportional to n^2A^2. So 1mw He-Ne laser is highly intense than the sun.

4. Highly coherence

A predictable correlation of the amplitude and phase at any one point with other point is called coherence.In case of conventional light, the property of coherence exhibits between a source and its virtual source where as in case of laser the property coherence exists between any two sources of same phase.

There are two types of coherence

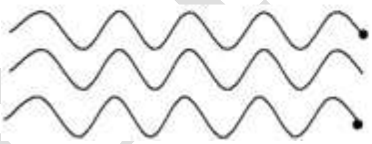
i) Temporal coherence
ii) Spatial coherence.

Temporal coherence (or longitudinal coherence):

The predictable correlation of amplitude and phase at one point on the wave train w .r. t another point on the same wave train, then the wave is said to be temporal coherence.

Spatial coherence (or transverse coherence):

The predictable correlation of amplitude and phase at one point on the wave train w. r .t another point on a second wave, then the waves are said to be spatial coherence (or transverse coherence).Two waves are said to be coherent when the waves must have same phase & amplitude.



INTERACTION OF LIGHT WITH MATTER AND THE THREE QUANTUM PROCESSES:

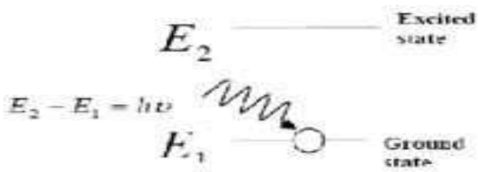
When the radiation interacts with matter, results in the following three important phenomena. They are

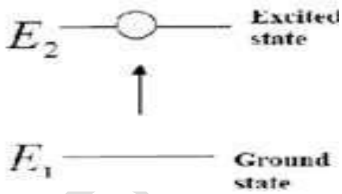
(i)Induced or Stimulated Absorption
(ii)Spontaneous Emission
(iii)Stimulated Emission

STIMULATED ABSORPTION (OR) INDUCED ABSORPTION (OR) ABSORPTION:

An atom in the lower energy level or ground state energy level (E_1) absorbs the incident photon and goes to excited state (E_2) as shown in figure below. This process is called induced or stimulated absorption.

Let E_1 and E_2 be the energies of ground and excited states of an atom. Suppose, if a photon of energy $E_2 - E_1 = h\nu$ interacts with an atom present in the ground state, the atom gets excitation form ground state E_1 to excited state E_2 . This process is called stimulated absorption.





Stimulated absorption rate depends upon the number of atoms available in the lowest energy state as well as the energy density of photons.

SPONTANEOUS EMISSION:

The atom in the excited state returns to ground state emitting a photon of energy (E) = $E_2 - E_1 = h\nu$, without applying an external energy spontaneously is known as spontaneous emission.

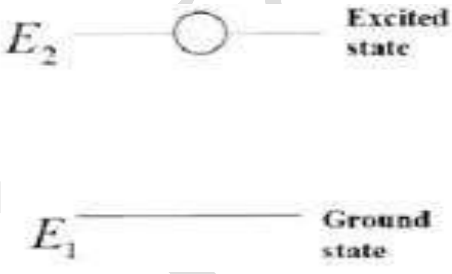
Let E_1 and E_2 be the energies of ground and excited states of an atom. Suppose, if photon of energy $E_2 - E_1 = h\nu$ interacts with an atom present in the ground state, the atom gets excitation form ground state E_1 to excited state E_2

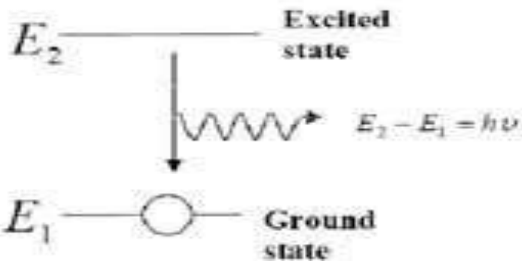
The excited atom does not stay for a long time in the excited state. The excited atom gets de-excitation after its life time by emitting a photon of energy $E_2 - E_1 = h\nu$. This process is called spontaneous emission.

The spontaneous emission rate depends up on the number of atoms present in the excited state.

The probability of spontaneous emission (P_{21}) is independent of $u(\vartheta)$.

$$P_{21} = A_{21}$$





STIMULATED-EMISSION:

The atom in the excited state can also returns to the ground state by applying external energy or inducement of photon thereby emitting two photons which are having same energy as that of incident photon. This process is called as stimulated emission.

Stimulated emission was postulated by Einstein. Let E_1 and E_2 be the energies of ground and excited states of an atom. Let a Photon of energy $E_2 - E_1 = h\upsilon$ interacts with the excited atom with in their life time The atom gets de-excitation to ground state by emitting of another photon.

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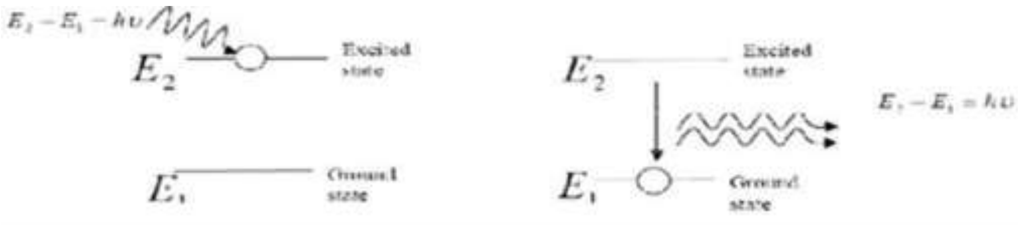
These photons have same phase and it follows coherence. This phenomenon is called stimulated emission



Stimulated emission rate depends upon the number of atoms available in the excitedstate as well as the energy density of photons.

Comparison between Spontaneous and Stimulated emission:

Spontaneous emission	Stimulated emission
1. The spontaneous emission Was Postulated by Bohr.	1.The stimulated emission was Postulated by Einstein.
2. Additional photons are not required in spontaneous emission.	2.Additional photons are required in Stimulated emission.
3. One Photon is emitted in spontaneous emission.	3. Two photons are emitted in stimulated emission.
4. The emitted radiation is incoherent.	5. The emitted radiation is coherent.
5. The emitted radiation is less intense.	6. The emitted radiation is high intense.

Light Amplification:

Light amplification requires stimulated emission exclusively. In practice, absorption and spontaneous emission always occur together with stimulated emission. The laser operation is achieved when stimulated emission exceeds the other two processes due to its higher transitions rates of atomic energy levels.

EINSTEIN COEFFICIENTS & THEIR RELATIONS

Einstein coefficients:

Einstein mathematically expressed the statistical nature of the three possible radiative transition routes (spontaneous emission, stimulated emission, and absorption) with the so-called Einstein coefficients and quantified the relations between the three processes.

Let N_1 be the number of atoms per unit volume with energy E_1 and $\mathbf{N_2}$ be the number of atoms per unit volume with energy E_2. Let 'n' be the number of photons per unit volume at frequency 'υ' such that $E_2 - E_1 = h\vartheta$ Then, the energy density of photons E= h .

- **Stimulated absoption**: When the photons interact with the atoms it leads to absorption transition which is called as stimulated absorption. Stimulated absorption rate depends upon the number of atoms available in the lowest energy state as well as the energy density photons.

  Stimulated absorption rate $\propto N_1$

  $\propto u(\vartheta)$

  $$= B_{12} N_1 u(\vartheta)$$

  Where $\mathbf{B_{12}}$ is the Einstein coefficient of stimulated absorption.

- **Spontaneous emission**: The atom in the excited state returns to ground state emitting a photon of energy (E) = $E_2 - E_1$ =hv, without applying an external energy spontaneously is known as spontaneous emission. The spontaneous emission rate depends up on the number of atoms present in the excited state.

  Spontaneous emission rate $\propto N_2$

  $$= A_{21} N_2$$

Where A21 is the Einstein coefficient of spontaneous emission.

- **Stimulated emission**: The atom in the excited state can also returns to the ground state by applying external energy or inducement of photon thereby emitting two photons which are having same energy as that of incident photon. This process is called as stimulated emission. Stimulated emission rate depends upon the number of atoms available in the excited state as well as the energy density of incident photons.

  Stimulated emission rate $\propto N_2$

  $\propto u(\vartheta)$

  $$= B_{21} N_2 u(\vartheta)$$

  Where $\mathbf{B_{21}}$ is the Einstein coefficient of stimulated emission.

If the system is in equilibrium, the rates of absorption transitions (Stimulated absorption) are equal to emission transitions (The rate of Spontaneous + Stimulated emission).

The rate of Absorption = The rate of Spontaneous + Stimulated emission

$$B_{12}N_1u(\vartheta) = A_{21}N_2 + B_{21}N_2u(\vartheta)$$

$$B_{12}N_1u(\vartheta) - B_{12}N_2u(\vartheta) = A_{21}N_2$$

$$u(\vartheta)(B_{12}N_1 - B_{21}N_2) = A_{21}N_2$$

$$u(\vartheta) = \frac{A_{21N_2}}{B_{12N_1 - B_{21}N_2}}$$

$$u(\vartheta) = (\frac{A_{21}}{B_{21}} \times \frac{1}{\left[\frac{N_1}{N_2} \times \frac{B_{12}}{B_{21}} - 1\right]})$$

According to Boltzmann distribution law

$$N = N_0 e^{-\frac{E}{k_B T}}$$

$$N_1 = N_0 e^{-\frac{E_1}{k_B T}}$$

$$N_2 = N_0 e^{-\frac{E_2}{k_B T}}$$

$$\frac{N_1}{N_2} = \frac{N_0 e^{-\frac{E_1}{k_B T}}}{N_0 e^{-\frac{E_2}{k_B T}}} = e^{\frac{E_2 - E_1}{k_B T}} = e^{\frac{h\vartheta}{k_B T}}$$

$$u(\vartheta) = \frac{A_{21}}{B_{21}} \times \frac{1}{[e^{\frac{h\vartheta}{k_B T}}\left(\frac{B_{12}}{B_{21}}\right) - 1]} \text{-----}(1)$$

According to Planck's radiation formula the energy density of photon is given by,

$$u(\vartheta) = \frac{8\pi h\vartheta^3}{c^3} \times (\frac{1}{\left[e^{\frac{h\vartheta}{k_B T}} - 1\right]}) \text{-----}(2)$$

Comparing eq.(1) & eq.(2), we get

$$(i) \qquad \frac{A_{21}}{B_{21}} = \frac{8\pi h\vartheta^3}{c^3} \quad or \quad \frac{A_{21}}{B_{21}} \propto \vartheta^3$$

$$(ii) \qquad \frac{B_{12}}{B_{21}} = 1 \quad or, \quad B_{12} = B_{21}$$

POPULATION INVERSION:

The number of atoms present in the excited (or higher) state is greater than the number of atoms present in the ground energy state (or lower state) is called population inversion.

Let us consider two level energy systems of energies $E_1$and E_2as shown in figure. Let N_1 and N_2 be the population (means number of atoms per unit volume) of $E_1$and E_2 respectively.

According to Boltzmann's distribution the population of an energy level E, at temperature T is given by

$$N_i = N_o^{\left(-\frac{E}{_BT}\frac{}{i}\right)} where i = 1,2,3, \dots . N_i$$

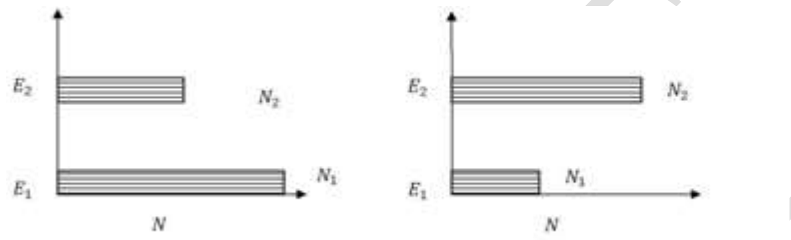
where 'N_0' is the number of atoms in ground or lower energy states & k is the Boltzmann constant.

From the above equation the population of energy levels E_1& E_2 are given by

$$N_1 = N_o e^{\left(-\frac{E_1}{k_B T}\right)}$$

$$N_2 = N_o e^{\left(-\frac{E_2}{k_B T}\right)}$$

At ordinary conditions $N_1 > N_2$ i.e., the population in the ground or lower state is always greater than the population in the excited or higher states. The stage of making, population of higher energy level greater than the population of lower energy level is called population inversion i.e., $N_2 > N_1$.



At normal conditions $N_1 > N_2$ After population inversion is achieved $N_2 > N_1$

METASTABLE STATE: In general the number of excited particles in a system is smaller than the non excited particles. The time during which a particle can exist in the ground state is unlimited. On the other hand, the particle can remain in the excited state for a limited time known as life time.

The life time of the excited hydrogen atom is of the order of 10^{-8}sec. However there exist such excited states in which the life time is greater than 10^{-8}sec. These states are called as Meta stable states.

PUMPING MECHANISMS (OR TECHNIQUES):

Pumping:

The process of rising more no of atoms to the excited state by artificial means is called pumping.

A system in which population inversion is achieved is called as an active system. The method of raising the particles from lower energy state to higher energy state is called pumping. (or the process of achieving of population inversion is called pumping). This can be done by number of ways. The most commonly used pumping methods are

- Optical pumping
- Electrical discharge pumping
- Chemical pumping
- Injection current pumping

Optical pumping:

Optical pumping is used in solid laser. Xenon flash tubes are used for optical pumping. Since these materials have very broad band absorption, sufficient amount of energy is absorbed from the emission band of flash lamp and population inversion is created. Examples of optically pumped lasers are ruby, Nd: YAG Laser (Y_3 AL_5G_{12})
(Neodymium: Yttrium Aluminum Garnet), Nd: Glass Laser

Electrical discharge pumping:

Electrical discharge pumping is used in gas lasers. Since gas lasers have very narrow absorption band pumping them any flash lamp is not possible. Examples of Electrical discharge pumped lasers are He-Ne laser, CO_2 laser, argon-ion laser, etc

Chemical pumping:

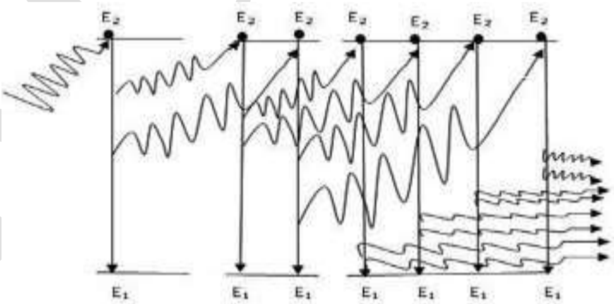
Chemical reaction may also result in excitation and hence creation of population inversion in few systems. Examples of such systems are HF and DF lasers.

Injection current pumping:

In semiconductors, injection of current through the junction results in creates of population inversion among the minority charge carriers. Examples of such systems are InP and GaAs.

PRINCIPLE OF LASER/LASING ACTION:

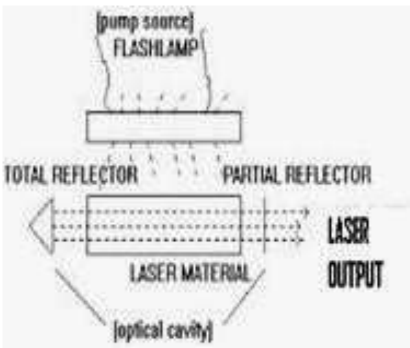
Let us consider many no of atoms in the excited state. Now the stimulating photon interacts with any one of the atoms in the excited state, the stimulated emission will occur. It emits two photons, having same energy & same frequency move in the same direction. These two photons will interact with another two atoms in excited state & emit 8-photons. In a similar way chain reaction is produced this phenomenon is called “Principle of lasing –action”. We get a monochromatic, coherent, directional & intense beam is obtained. This is called laser beam. This is the principle of working of a laser.



Components of a LASER:

Any laser system consists of 3-important components. They are

(i) Source of energy or pumping source
(ii) Active-medium (Laser Material)
(iii)Optical cavity or resonator



(i) Energy Source : It supply energies & pumps the atoms or molecules in the active medium to excited states. As a result we get population inversion in the active medium which emits laser.

Ex: Xenon flash lamp, electric field.

(ii) Active medium: The medium in which the population inversion takes place is called as active medium.

Active-centre: The material in which the atoms are raised to excited state to achieve population inversion is called as active center.

(iii) Optical-cavity or resonator: The active medium is enclosed between a fully reflected mirror & a partially reflective mirror. This arrangement is called as cavity or resonator. As a result, we get highly intense monochromatic, coherence laser light through the non-reflecting portion of the mirror.

DIFFERENT TYPES OF LASERS

On the basis of active medium used in the laser systems, lasers are classified into several types

I. Solid lasers : Ruby laser, Nd;YAG laser, Nd;Glass
II. Liquid lasers : Europium Chelate laser, $SeOCl_2$
III. Gas lasers : CO_2, He-Ne, Argon-Ion Laser
IV. Dye lasers : Rhodamine 6G
V. Semiconductor lasers : InP, GaAs.

RUBY LASER

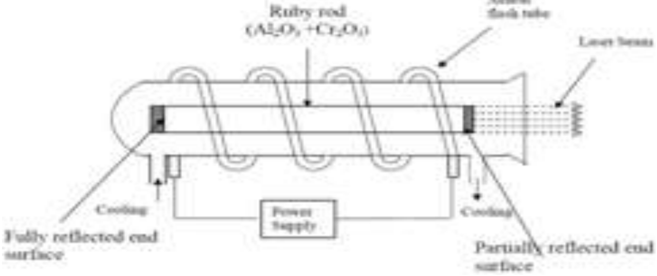
Ruby laser is a three level solid state laser and was developed by Mainmann in 1960. Ruby (Al_2O_3+Cr_2O_3) is a crystal of Aluminium oxide, in which 0.05% of Al^{+3} ions are replaced by the Cr^{+3} ions. The colour of the ruby rod is pink. The active medium in the ruby rod is Cr^{+3} ions.

Principle or Characteristics of a ruby laser:

Due to optical pumping, the chromium atoms are raised to excited states then the atoms come to metastable state by non-radiative transition. Due to stimulated emission the transition of atoms takes place from metastable state to ground state and gives a laser beam.

Construction:

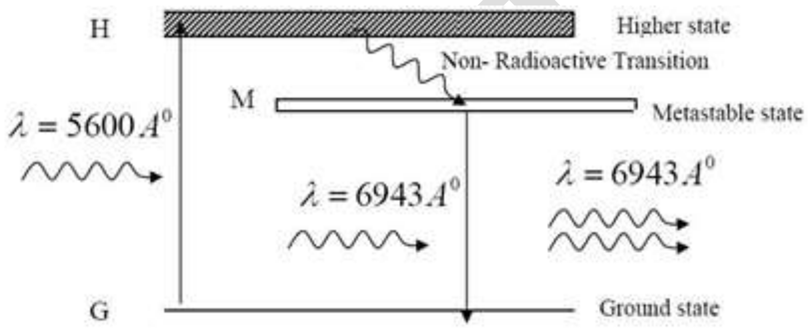
- In ruby laser 4cm length and 5mm diameter rod is generally used.
- Both the ends of the rods are highly polished and made strictly parallel.
- The ends are silvered in such a way, one becomes partially reflected and the other end fully reflected.
- The ruby rod is surrounded by xenon flash tube, which provides the pumping light to excite the chromium ions in to upper energy levels.



- Xenon flash tube emits thousands joules of energy in few milli seconds, but only a part of that energy is utilized by the chromium ions while the rest energy heats up the apparatus.
- A cooling arrangement is provided to keep the experimental set up at normal temperatures.

Working:

- The energy level diagram of chromium ions is shown in figure.
- The chromium ions get excitation into higher energy levels by absorbing of $5600A^0$ of wave length radiation.
- The excited chromium ions stay in the level H for short interval of time (10^{-8} Sec).
- After their life time most of the chromium ions are de-excited from H to G and a few chromium ions are de-excited from H to M.



- The transition between H and M is non-radioactive transition i.e. the chromium ions gives their energy to the lattice in the form of heat.
- In the Meta stable state the life time of chromium ions is 10^{-3} sec.
- Due to the continuous working of flash lamp, the chromium ions are excited to higher state H and returned to M level.
- After few milli seconds the level M is more populated than the level G and hence the desired population inversion is achieved.
- The state of population inversion is not a stable one.
- The process of spontaneous transition is very high.
- When the excited chromium ion passes spontaneously from H to M it emits one photon of wave length $6943A^0$.
- The photon reflects back and forth by the silver ends and until it stimulates an excited chromium ion in M state and it to emit fresh photon in phase with the earlier photon.
- The process is repeated again and again until the laser beam intensity is reached to a sufficient value.
- When the photon beam becomes sufficient intense, it emerges through the partially silvered end of the rod.
- The wave length $6943A^0$ is in the red region of the visible spectrum on returning to ground state (G).

Uses of Ruby laser:

- Used in distance measurement using ‘pulse echo technique’
- Used for measurement of plasma properties such as electron density and temperature.
- Used to remove the melanin of the skin.
- Used for recording pulsed holograms.
- Used as target designators and range finders in military.

Draw backs of Ruby laser:

- It requires high pumping power.
- The efficiency of ruby laser is very small. It is a pulse laser.

He-Ne LASER

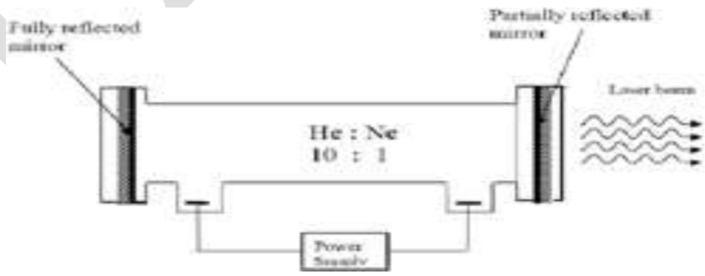
It was discovered by A. Javan & his co-workers in 1960. It is a continuous wave gas laser. It consists of mixture of He & Ne in 10:1 ratio as a active medium.

Principle/Characteristics of He-Ne laser:

This laser is based on the principle of stimulated emission, produced in the He & Ne. The population inversion is achieved due to the interaction between He & Ne gases.Using gas lasers, we can achieve highly coherent, directional and high monochromatic beam.

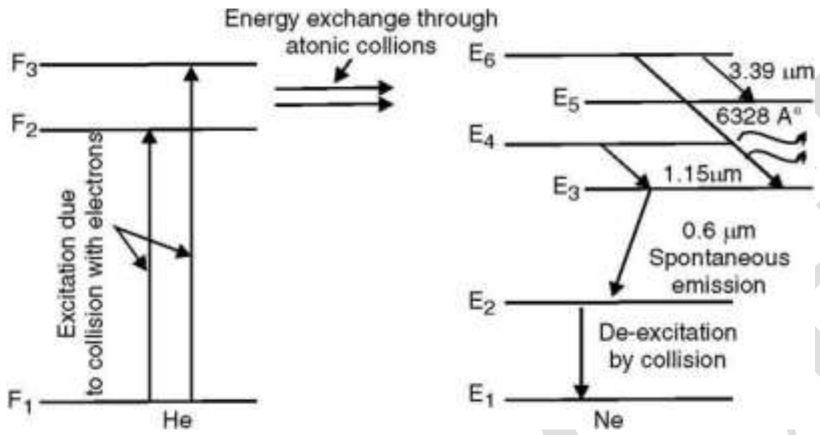
Construction:

- In He-Ne gas laser, the He and Ne gases are taken in the ratio 10:1 in the discharge tube.
- Two reflecting mirrors are fixed on either ends of the discharge tube, in that, one is partially reflecting and the other is fully reflecting.
- In He-Ne laser 80cm length and 1cm diameter discharge tube is generally used.
- The output power of these lasers depends on the length of the discharge tube and pressure of the gas mixture.
- Energy source of laser is provided by an electrical discharge of around 1000V through an anode and cathode at each end of the glass tube.



Working:

- When the electric discharge is passing through the gas mixture, the electrons accelerated towards the positive electrode.
- During their passage, they collide with He atoms and excite them into higher levels.
- F_2 and F_3 form F_1. In higher levels F_2 and F_3, the life time of He atoms is more.
- So there is a maximum possibility of energy transfer between He and Ne atoms through atomic collisions.
- When He atoms present in the levels F_2 and F_3 collide with Ne atoms present ground state E_1, the Ne atoms gets excitation into higher levels E_4 andE_6.



- Due to the continuous excitation of Ne atoms, we can achieve the population inversion between the higher levels E_4 (E_6) and lower levels E_3 (E_5).
- The various transitions $E_6 \rightarrow E_5$, $E_4 \rightarrow E_3$ and $E_6 \rightarrow E_3$ leads to the emission of wavelengths 3.39A^0, 1.15 A^0 and 6328A^0.
- The first two corresponding to the infrared region while the last wavelength is corresponding to the visible region.
- The Ne atoms present in the E_3 level are de-excited into E_2 level, by spontaneously emission of photon.
- When a narrow discharge tube is used, the Ne atoms present in the level E_2 collide with the walls of the tube and get de-excited to ground level E_1.

Uses of He-Ne laser:

- Used in laboratories foe all interferometric experiments.
- Used widely in metrology in surveying, alignment etc.
- Used to read barcodes and He-Ne laser scanners also used for optical character recognition.
- Used in holography.

CO2 laser :

The CO2 laser was one of the gas laser,these are the highest power continuous wave lasers that are currently available,they are also quite efficient .The ratio of output power to pump power can be as large as 20%.

The CO2 laser produces a beam of infrared light with the principle wavelength bands centering around 9.6 and 10.6 µm.

Principle: The principle of carbon di oxide laser is transition between vibrational states of the same electronic states by achieving population inversion between these states.

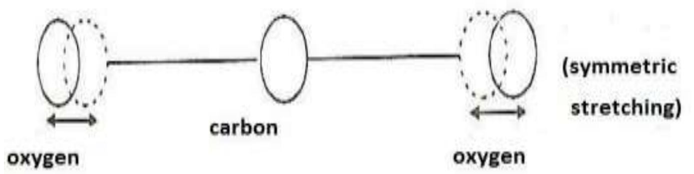
For the laser action two points are important ,one is the population inversion between the two levels and second increased density of the incident radiation.

Vibrational states in co2 molecules:

A carbon dioxide molecule has a carbon atom at the centre with two oxygen atoms attached, one at both sides. Such a molecule exhibits three independent modes of vibrations. They are

a) Symmetric stretching mode.
b) Bending mode
c) Asymmetric stretching mode.

a) **Symmetric stretching mode :** In this mode of vibration, carbon atoms are at rest and both oxygen atoms vibrate simultaneously along the axis of the molecule departing or approaching the fixed carbon atoms



b) **Bending mode:**

In this mode of vibration, oxygen atoms and carbon atoms vibrate perpendicular to molecular axis.

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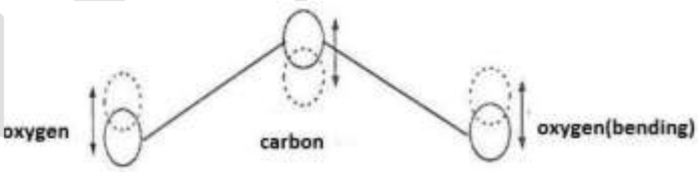
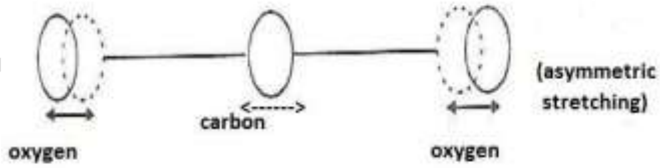
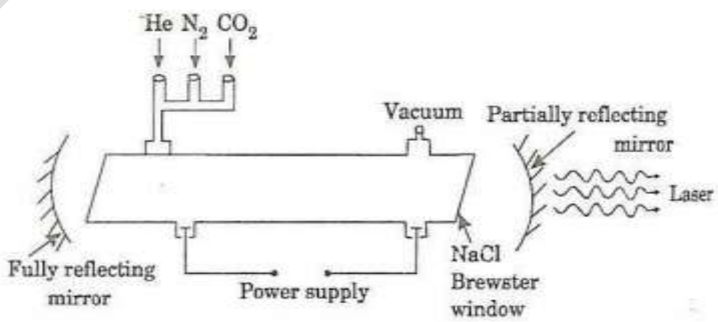


c) **Asymmetric Stretching Mode :**

In this mode of vibration, oxygen atoms and carbon atoms vibrate asymmetrically, i.e., oxygen atoms move in one direction while carbon atoms in the other direc



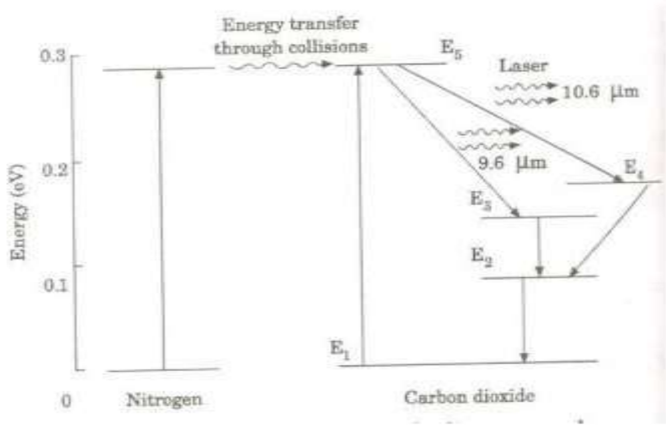
Consturction:



- It consists of a quartz tube 5 m long and 2.5 cm in the diameter.
- This discharge tube is filled with gaseous mixture of CO2 (active medium), helium and nitrogen with suitable partial pressures.
- The terminals of the discharge tubes are connected to a D.C power supply.
- The ends of the discharge tube are fitted with NaCl Brewster windows so that the laser light generated will be polarized.
- Two concave mirrors one fully reflecting and the other partially form an optical resonator.

Working :

Figure shows energy levels of nitrogen and carbon dioxide molecules.



- A mixture of CO2 , N2 and helium or water vapour is used as active medium here.
- When an electric discharge occurs in the gas, the electrons collide with nitrogen molecules and they are raised to excited states. This process is represented by the equation.

  N2 + e* = N2* + e
  N2 = Nitrogen molecule in ground state e* = electron with kinetic energy
  N2* = nitrogen molecule in excited state e= same electron with lesser energy
- Now N2 molecules in the excited state collide with CO2 atoms in ground state and excite to higher electronic, vibrational and rotational levels. This process is represented by the equation

  N2* + CO2 = CO2* + N2
  N2* = Nitrogen molecule in excited state.
  CO2 = Carbon dioxide atoms in ground state
  CO2* = Carbon dioxide atoms in excited state
  N2 = Nitrogen molecule in ground state.

- Since the excited level of nitrogen is very close to the E5 level of CO2 atom, population in E5 level increases.
- As soon as population inversion is reached, any of the spontaneously emitted photon will trigger laser action in the tube. There are two types of laser transition possible.
- **1.Transition E5 to E4** : This will produces far IR radiation of laser beam at the wavelength 10.6µm
- **2.Transition E5 to E3:**This laser transition will produce a laser beam of wavelength 9.6µm in IR region. Normally 10.6µm transition is more intense than 9.6µm transition. The power output from this laser is 10kW.

E3 and E4 levels also metastable states and the co_2 molecules at these levels fall to the lower level E2 through inelastic collisions with unexited co_2 molecules, then again tends to closer ground state through thermal excitations.The nature of output may be continuous wave or pulsed wave.

Advantages:

1. The construction of CO2 laser is simple and the output of this laser is continuous.
2. It has high efficiency
3. It has very high output power.
4. The output power can be increased by extending the length of the gas tube.

Disadvantages:

1. The contamination of oxygen by carbon monoxide will have some effect on laser action
2. The operating temperature plays an important role in determining the output power of laser.
3. The corrosion may occur at the reflecting plates.
4. Accidental exposure may damage our eyes, since it is invisible (infrared region) to our eyes.

Applications:

1. High power CO2 laser finds applications in material processing, welding, drilling, cutting soldering etc.
2. The low atmospheric attenuation (10.6µm makes CO2 laser suitable for open air communication.
3. It is used for remote sensing.
4. It is used for treatment of liver and lung diseases.
5. It is mostly used in neuro surgery and general surgery.
6. It is used to perform microsurgery and bloodless operations.

Semiconductor laser

The semiconductor laser is also called diode laser. Among the different semiconductors there are direct band gap semiconductors and indirect band gap semiconductors. In the case of direct band gap semiconductors, there is a large possibility for direct recombination of holes and electrons which emit photons. But in indirect band gap semiconductors like silicon and germanium, direct recombination

of holes and electrons is less possible and hence there is no effective emission of photons. A well know example of a direct band gap semiconductor is GaAs. Let E g be the energy gap of a material, then it emits a photon of wavelength (λ) is given the relation

$$\lambda = \frac{Ch}{Eg} \quad \rightarrow (1)$$

Where C is the velocity of light and h is the Planck's constant
The equation (1) is reduced to

$$\lambda = \frac{1.24}{Eg}\ \mu m \quad \rightarrow (2)$$

Where E_g is expressed in eV

As E_g increases, it emits shorter wavelengths. Diode lasers are always operated in forward bias. If p and n type materials are prepared from the same material then the p-n junction, is called as Homo junction semiconductor laser source. If p and n type materials are prepared from different materials then they are called as Hetero junction semiconductor laser source.

Construction of GaAs Semiconductor diode laser

The basic mechanism responsible for light emission from a semiconductor is the recombination of electrons and holes at p-n junction when a current is passed through the diode. The active medium is a p-n junction diode made from crystalline Gallium Arsenide. The p-region and n-region in the diode are obtained by heavily doping with suitable do pants. At the junction the sides through which emitted light is coming out are well polished and parallel to each other. Since the refractive index of GaAs is high, the reflectance at the material air interface is sufficiently large so that the external mirrors are not necessary to produce multiple reflections. When a current is passed through a p-n junction p region being positively biased, holes are injected from p-region into n-region and n-region being negatively biased electrons are injected from n-region into p-region and is shown in fig8.9.

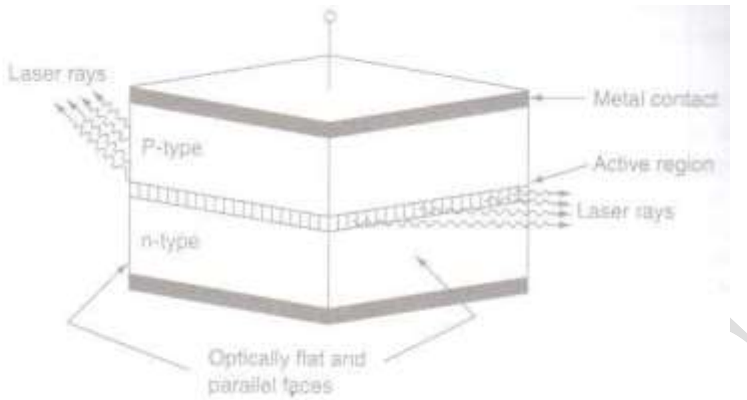
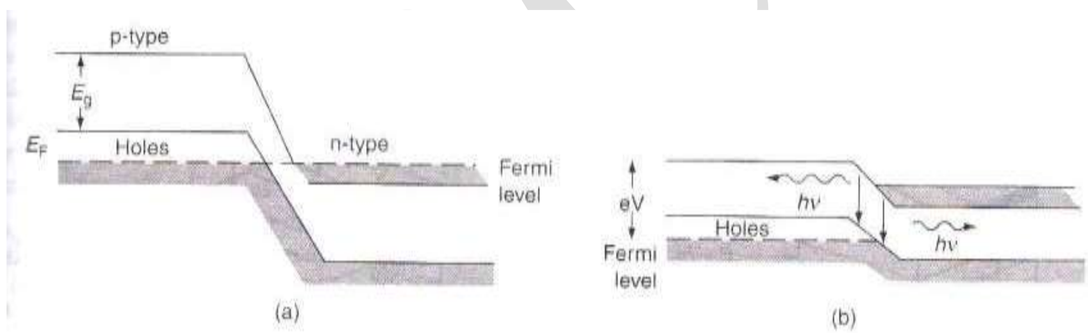


Fig.2.8 GaAs semiconductor diode laser

The connections in this p-n junction circuit are called forward bias. The electrons and holes recombine and release of light energy takes place in or near the junction. In the case of GaAs homo junction which has an energy gap of 1.44ev gives a laser beam of wavelength around 8600A^0 the electron-hole recombination takes place in the active region of the device. When the junction is forward biased, a large amount of the order of 104 amp/cm^2 is passed through the narrow junction. Thus the electrons (holes) are injected from n side to p side (p side to n side) respectively.



Fi2.9 (a) Band diagram of heavily doped p-n junction in equilibrium.

(b) Band diagram of heavily doped p-n junction with forward bias

The electrons are minority charge carriers in p-side and holes are the minority charge carriers in n-side. The continuous injection of charge carriers creates the population inversion of minority carriers is n and p side respectively. The excess minority charge carries diffuse away from the region recombining with majority carriers of the n and p type material, resulting in the release of photons. Further, the emitted photons increase the recombination of injected electrons from the n-region and holes in p-region by inducing more recombination thus the stimulated emission takes place more effectively. The efficiency of a laser emission increases when the device is cooled.

The drawbacks of homojunction lasers

1. Only output is obtained
2. The threshold current density is very large(400amps/mm^2)
3. Electromagnetic confinement is poor.
4. Output has more beam divergence, poor coherence.

To overcome these deficiencies, scientists have developed heterojunction laser sources like GaAs/GaAlAs structures. The advantages of heterojunction laser structures are:

1. Low threshold current density(5-10amp/mm^2)
2. The output is continuous
3. Carrier confinement is more effective there by less beam divergence.
4. High output power.
5. Narrow beam, high coherence, high mono chromaticity is achieved.
6. Long lifetime of the device
7. Very stable. Hence hetero junction laser diode used extensively in optical fiber communication

BAR CODE SCANNER

A barcode scanner is a device that reads printed barcodes and converts them into digital data that a computer or system can understand. Barcodes store information such as product numbers, serial numbers, or other identifiers in a series of parallel lines (bars) and spaces.

Steps involved in the Bar code scanner working:

1. Scanning: The scanner emits a light beam (usually a laser or LED) onto the barcode.

2. Reflection: The light reflects back from the barcode to the scanner. Dark bars absorb light, and white spaces reflect it.

3. Detection: A sensor in the scanner detects the reflected light intensity.

4. Decoding: The scanner converts the light pattern into an electrical signal, then translates that into digital data (usually the numbers or characters encoded in the barcode).

5. Output: The decoded data is sent to a computer, cash register, or database system for processing.

Types of Barcode Scanners

Laser Scanners: Use a laser beam to scan the barcode.

CCD (Charge-Coupled Device) Scanners: Use an array of tiny light sensors.

Camera-Based Scanners (Imagers): Use a camera to capture an image and software to decode the barcode.

Pen-type Scanners: Manually dragged across the barcode.

Applications of Barcode Scanners

1. Retail and Point of Sale (POS)

Speed up checkout by scanning product barcodes.

Inventory management and stock control.

2. Warehouse and Inventory Management

Track goods in storage.

Manage shipments and deliveries.

3. Healthcare

Patient identification.

Medication tracking and administration.

4. Manufacturing

Track parts and assembly line management.

5. Libraries

Check-in/check-out of books and materials.

6. Logistics and Shipping

Track packages and shipments.

7. Event Management

Scan tickets and passes.

LIDAR FOR AUTONOMOUS VEHICLE

LIDAR stands for Light Detection and Ranging. It's a remote sensing technology that uses laser light to measure distances to objects. By emitting rapid laser pulses and measuring the time it takes for the reflections to return, LIDAR creates detailed, precise 3D maps of the environment. Essentially, it works like radar but uses light instead of radio waves. It generates a point cloud—a collection of spatial points that represent the surroundings.

Applications of LIDAR for Autonomous Vehicles

1. Environment Mapping and 3D Perception:

LIDAR creates a real-time 3D map of the car's surroundings, identifying road edges, obstacles, and terrain.

2. Obstacle Detection and Avoidance:

Detects other vehicles, pedestrians, cyclists, and road hazards with high accuracy and range.

Helps the vehicle plan safe maneuvers.

3. Localization and Navigation:

Works with GPS and other sensors to pinpoint the vehicle's exact position in the environment.

Allows autonomous vehicles to understand their location relative to the road and nearby objects.

4. Lane Detection and Road Feature Recognition:

Identifies lane markings, curbs, traffic signs, and traffic lights for better decision-making.

5. Adaptive Cruise Control and Collision Avoidance:

Continuously monitors traffic to maintain safe distances and avoid collisions.

APPLICATIONS OF LASERS

Due to high intensity, high mono-chromaticity and high directionality of lasers, they are widely used in various fields like

1. communication
2. computers
3. chemistry
4. photography
5. industry
6. medicine
7. military
8. scientific research

1. Communication:

- In case of optical communication, semiconductors laser diodes are used as optical sources.
- More channels can be sent simultaneously Signal cannot be tapped as the band width is large, more data can be sent.
- A laser is highly directional and less divergence, hence it has greater potential use in space crafts and submarines.

2. Computers :

- In LAN (local area network), data can be transferred from memory storage of one computer to other computer using laser for short time.
- Lasers are used in CD-ROMS during recording and reading the data.

3. Chemistry :

- Lasers are used in molecular structure identification.
- Lasers are also used to accelerate some chemical reactions.
- Using lasers, new chemical compound can be created by breaking bonds between atoms or molecules.

4. Photography :

- Lasers can be used to get 3-D lens less photography.
- Lasers are also used in the construction of holograms.

5. Industry :

- Lasers can be used to blast holes in diamonds and hard steel.
- Lasers are also used as a source of intense heat.
- Carbon dioxide laser is used for cutting drilling of metals and nonmetals, such as ceramics plastics, glass etc.
- High power lasers are used to weld or melt any material.
- Lasers are also used to cut teeth in saws and test the quality of fabric.

6. Medicine :

- Pulsed neodymium laser is employed in the treatment of liver cancer.
- Argon and carbon dioxide lasers are used in the treat men of liver and lungs.
- Lasers used in the treatment of Glaucoma.

7. Military :

- Lasers can be used as a war weapon.
- High energy lasers are used to destroy the enemy air-crafts and missiles.
- Lasers can be used in the detection and ranging likes RADAR.

8. **Scientific research:**

- Lasers are used in the field of 3D-photography.
- Lasers used in Recording and reconstruction of hologram.
- Lasers are employed to create plasma.
- Lasers used to produce certain chemical reactions.
- Lasers are used in Raman spectroscopy to identify the structure of the molecule.
- Lasers are used in the Michelson- Morley experiment.
- A laser beam is used to confirm Doppler shifts in frequency for moving objects.

FIBER-OPTICS

INTRODUCTION TO OPTICAL FIBER:

Fiber optics is a branch of physics which deals with the transmission & reception of light waves using optical fibers which acts as a guiding media. The transmission of light waves by fiber optics was first demonstrated by John Tyndall in 1870.

Optical Fiber:

Optical fiber is a thin & transparent guiding medium or material which guides the information carrying light waves. It is a cylindrical wave-guide system which propagates the data & speech signals in the optical frequency range.

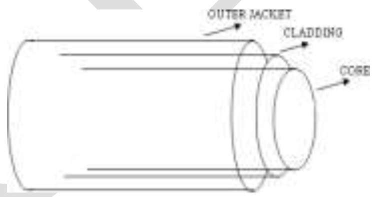
A light beam acting as a carrier wave is capable of carrying more information than radio waves & microwaves because of its high frequency as shown below.

Radio waves - 10^4 Hz , Micro waves - 10^{10} Hz, Light waves - 10^{15} Hz

Construction:

An optical fiber is a very thin, flexible transparent made with plastic or glass.

It has cylindrical shape consisting of three layers or sections



1) The Core
2) The Cladding
3) The Outer jacket or Buffer jacket

1) The Core: It is the central layer surrounded by another layer called cladding. Light is transmitted within the core which has refractive index (n_1). It is a denser medium. Core is made of silica (SiO_2).

2) The Cladding: It is the second layer, surrounded by a third layer called the outer jacket. It has refractive index n_2 which is less than the refractive index of core i.e ($n_1 > n_2$). It acts as a rarer medium. It keeps the light within the core because $n_1 > n_2$. To lower the refractive index of cladding the silica is doped with phosphorous or bismuth material.

3) The Outer or Buffer Jacket: It is the third layer it protects the fiber from moisture & abrasion. To provide necessary toughness & tensile strength, a layer of strength member is arranged surrounding buffer jacket. It is made of polyurethane material.

Working Principle of Optical Fiber:

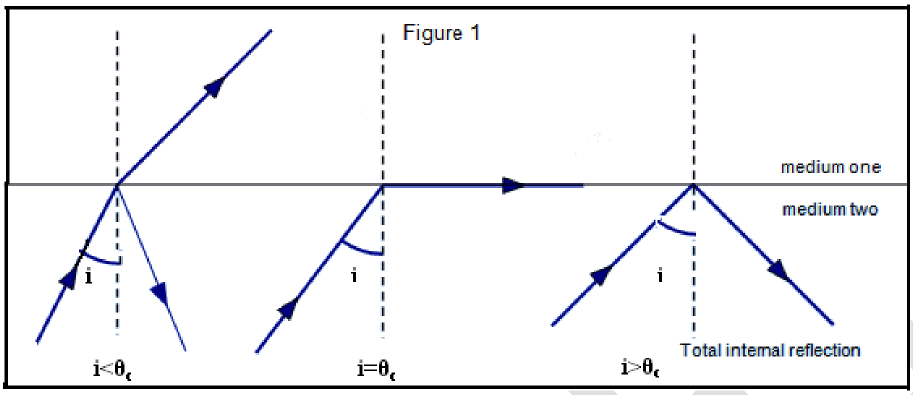
Total Internal Reflection: The principle of optical fiber is total internal reflection.

Condition for Total Internal Reflection:

1) The light ray should move from denser to rarer medium.
2) The refractive index of core must be greater than cladding i.e. $n_1 > n_2$
3) The angle of incidence (i) must be greater than the critical angle(θc) i.e. i>θc.
4) The critical angle $\theta_c = sin^{-1}\frac{n_2}{n_1}$.

Explanation:

Let us consider a denser medium & rarer medium of refractive indices n_1& n_2 respectively and $n_1>n_2$. Let a light ray move from denser to rare medium with 'i' as the angle of incidence & 'r' as angle of refraction. The refracted ray bends away from the normal as it travels from denser to rarer medium with increase of angle of incidence 'i'.



In this we get three cases

Case-1: When i<θc, Then the light ray refracts into rarer medium as shown in figure.

Case-2: When i= θc , then the light ray traverses along the two media as shown in figure.

For the two media, applying Snell's law

$$n_1 sini = n_2 sinr$$

$$(\therefore\ i = \theta_c \& r = 90°)$$

$$n_1 sin\theta_c = n_2 sin90$$

$$n_1 sin\theta_c = n_2$$

$$sin\,\theta_c = \frac{n_2}{n_1}$$

Where n1>n_2

For air, n_2 =1,

$$\theta_c = sin^{-1}\frac{1}{n_1}$$

Case-3: When i> θc, then the light ray reflected back into the medium as shown in figure

ACCEPTANCE ANGLE & ACCEPTANCE CONE:

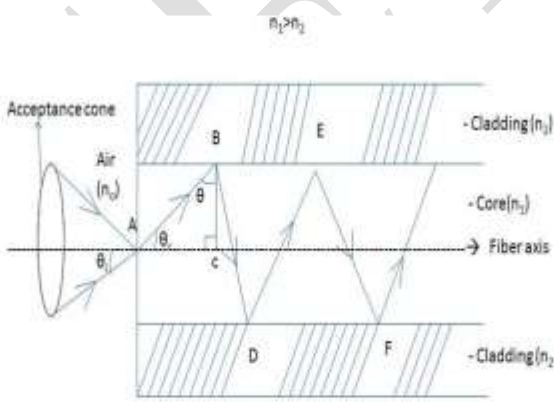
Def: Acceptance angle is the maximum angle of incidence at the core of an optical fiber so that the light can be guided though the fiber by total internal reflection. This angle is called as acceptance angle. It is denoted by 'α_i'.

- Consider a cross-sectional view of an optical fiber having core & cladding of refractive indices n_1 and n_2.
- Let the fiber be in air medium (n_o). The incident light while entering into the core at 'A' makes an incident angle of 'αi' with the fiber-axis.
- In core it travels along AB & is incident at part B on cladding interface.
- Let α_r be the angle of refraction at part 'A' & 'θ' be the angle of incidence at 'B'.
- When 'θ' is greater than the critical angle 'θc', then total internal reflection takes place into the core & light takes the path BD.
- Due to multiple total internal reflections the propagation of light ray takes place through the fiber.
- Applying Snell's law at AC core-air interface :-

$$\frac{\sin\alpha_i}{\sin\alpha_r} = \frac{n_1}{n_o}$$

$$n_o \sin\alpha_i = n_1 \sin\alpha_r \quad \rightarrow (1)$$

- Let a normal 'BC' be drawn from the point 'B' to the fiber axis. Then from ΔABC, we get



$$\alpha_r = 90° - \theta \rightarrow (2)$$

Substitute eq - (2) in eq - (1)

$$n_o \sin\alpha_i = n_1 si(90° - \theta)$$

$$n_o \sin\alpha_i = n_1 \cos\theta \rightarrow (3)$$

- To get total internal reflection at point B

(Core-Classing Interface) i.e. $\theta > \theta_c$

- Let the maximum angle of incidence at point A be α(max)for which $\theta > \theta_c$.

From eqn (3), we get

$$n_o \sin\alpha_i = n_1 \cos\theta \rightarrow (4)$$

$$\alpha(\max) = \alpha_i, \text{when } \theta = \theta_c$$

$$\sin \alpha_i(\max) = \frac{n_1}{n_o} \cos\theta_c \rightarrow (5)$$

We know that $\sin\theta_c = \frac{n_2}{n_1}$

$$\cos\theta_c = \sqrt{(1 - \sin\theta_c^2)} = \sqrt{(1 - \frac{n_2^2}{n_1^2})} => \sqrt{\frac{n_1^2 - n_2^2}{n_1^2}} = \frac{\sqrt{(n_1^2 - n_2^2)}}{n_1} \therefore \cos\theta_c = \frac{\sqrt{(n_1^2 - n_2^2)}}{n_1}$$

$\rightarrow (6)$

Substitute the eq(6) in eq(5) , we get

$$sin\ \alpha(\max) = \frac{n_1}{n_0}\sqrt{\frac{n_1^2 - n_2^2}{n_1}}$$

$$sin\ \alpha_i(\max) = \sqrt{\frac{n_1^2 - n_2^2}{n_0}}$$

(OR)

$$\alpha(\max) = sin^{-1}\sqrt{\frac{n_1^2 - n_2^2}{n_0}} \rightarrow (7)$$

For air medium, $n_0 = 1$ $\quad \alpha_i(\max) = sin^{-1}\sqrt{n_1^2 - n_2^2} \rightarrow (8)$

Fractional Index Change (Δ):

It is the ratio of refractive index difference in core & cladding to the refractive index of core.

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$n_1 - n_2 = \Delta n_1 \rightarrow (1)$

NUMERICAL APERTURE (N.A.):

Def: It is defined as light accepting efficiency of the fiber and is equal to sine of the acceptance angle of the fiber i.e. $N.A. = sin\ \alpha(\max)$

$$N.A = sin\ \alpha_{(\max)} = \sqrt{\frac{n_1^2 - n_2^2}{n_0}}$$

$$\text{We know that } \Delta = \frac{n_1 - n_2}{n_1}$$

$$n_1 - n_2 = \Delta n_1 \rightarrow (2)$$

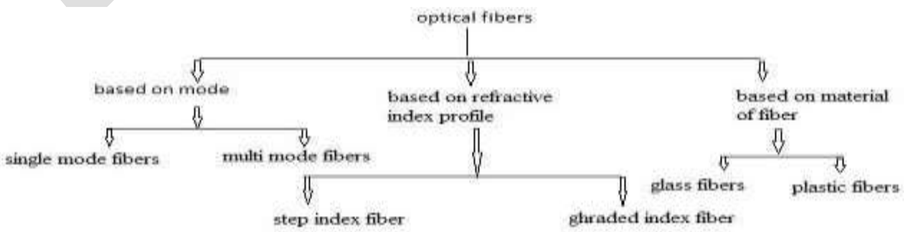
$$We know, N.A. = Sin\ \alpha(\max) = \sqrt{n_2^2 - n_1^2} \qquad \text{for air } n_0=1$$

$$N.A. = \sqrt{(n_1 + n_2)(n_1 - n_2)}$$

$$If n_1 = n_2, then N.A. = \sqrt{2n_1 \times \Delta n_1}$$

$$N.A. = n_1\sqrt{2\Delta} \rightarrow (3)$$

TYPES OF OPTICAL FIBERS:



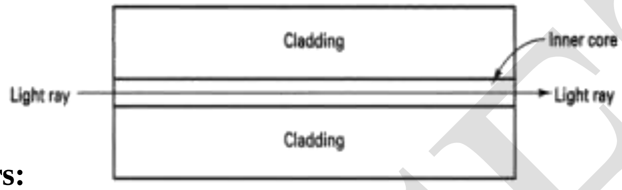
Optical fibers can be classified based on either the mode they support or the refractive index profile of the fiber. They can also be classified based on the material of the fiber.

Based on Mode:

The rays travelling in the fiber by total internal reflection are called modes.

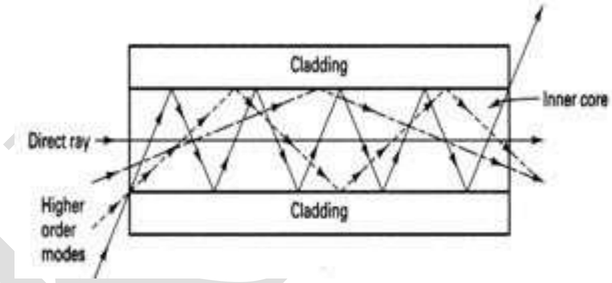
1) Single mode fibers:

- If the thickness of the fiber is so small that it supports only one mode then the fiber is called **single modefiber**or mono mode fiber.
- The core diameter of this fiber is about 8 to 10μm and the outer diameter of cladding is 60 to 70μm.



2) Multi mode fibers:

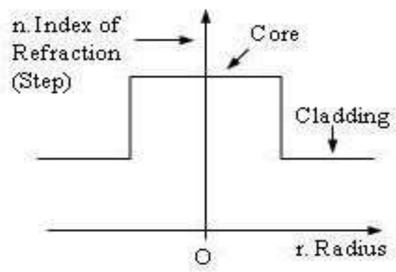
- If the thickness of the fiber is very large that it supports more than one mode then the fiber is called **multi mode fiber**.
- The core diameter of this fiber is about 50 to 200μm and the outer diameter of cladding is 100 to 250μm.



Based on refractive index profile:

1) Step-Index Optical fiber:

- In a step-index optical fiber, the entire core has uniform refractive index n_1 slightly greater than the refractive index of the cladding n_2.
- Since the index profile is in the form of a step, these fibers are called step-index fibers.
- The transmission of information will be in the form of signals or pulses.



- These are extensively used because distortion and transmission losses are very less.
- Step-index optical fibers are of two types.

They are

(i) Single mode step-index fiber

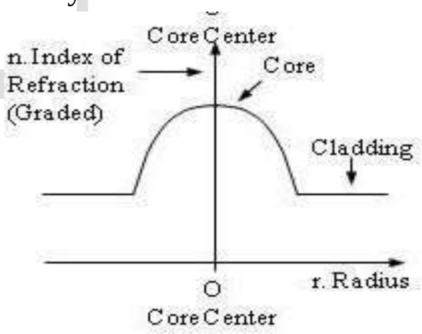
(ii) Multi-mode step-index fiber

Transmission / propagation of signal in Step-index fibers:

- Generally the signal is sent through the fiber in digital form i.e. in the form of pulses.
- The same pulsed signal travels in different paths.
- Let us now consider a signal pulse travelling through step index fiber in two different paths (1) and (2).
- The pulse (1) travelling along the axis of the fiber and pulse (2) travelling away from the axis.
- At the receiving end only the pulse (1)which travels along the fiber axis reaches first while the pulse(2) reaches after some time delay.
- Hence the pulsed signal received at the other end is broadened. This is called internal dispersion.
- This reduces transmission rate capacity of the signal.
- This difficulty is overcome by graded index fibers.

2) Graded index optical fiber:

- In this fiber, the refractive index of the core varies radially.
- It has maximum refractive index at its centre, which gradually falls with increase of radius and at the core-cladding interface matches with refractive index of cladding.
- Variation of refractive index of the core with radius is given by



$$n(x) = n_1 [1 - 2\Delta (\frac{x}{a})^p]^2$$

Where n_1 -> refractive index at the centre of the core

a-> radius of the core

Δ-> Fractional index change, $\Delta = \frac{n_1 - n_2}{n_1}$

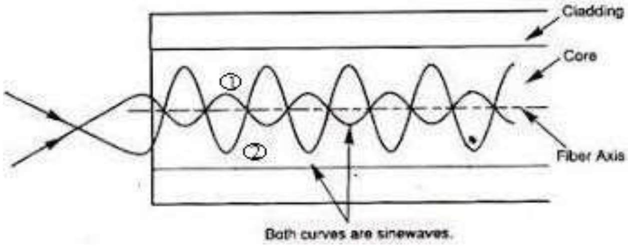
p-> grating profile index number

- This fiber divided into two types.

(i) Single-mode graded index fiber

(ii) Multi-mode graded index fiber

Transmission / propagation of signal in Step-index fibers:



- Let us now consider a signal pulse travelling through graded index fiber in two different paths (1) and (2).
- The pulse (1) travelling along the axis of the fiber though travels along shorter route it travels through higher refractive index.
- The pulse (2) travelling away from the axis undergo refraction and bend as shown in fig. though it travels longer distance, it travels along lesser refractive index medium.
- Hence both the pulses reach the other end simultaneously.
- Thus the problem of inter model dispersion can be overcome by using graded index fibers.

Based on types of materials:

1) glass-glass optical fiber
2) glass-plastic optical fiber
3) plastic-plastic optical fiber

ATTENUATION (POWER-LOSS) IN OPTICAL FIBERS

When light propagates through an optical fiber, then the power of the light at the output end is found to be always less than the power launched at the input end. The loss of power is called Attenuation. It is measured in terms of decibels per kilometer.

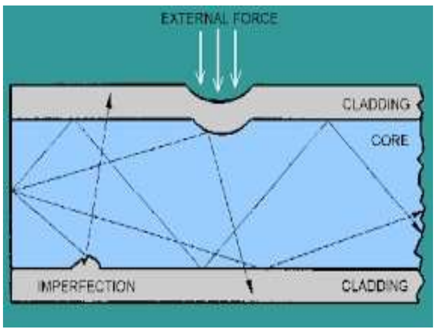
Attenuation: It is defined as the ratio of the optical power output (P_{out}) from a fiber of length 'L' to the power input (P_{in}).

$$Attenuatio(\alpha) = -\frac{10}{L}\left(\frac{P_{in}}{P_{out}}\right) dB/km$$

Attenuation occurs because of the following reasons

(1) Absorption (2) Scattering loss 3) Bending loss

(1) Absorption:



- It occurs in two ways, i)Absorption by impurity or impurity absorption
  ii) Intrinsic absorption or internal absorption
- **Impurity absorption:** The impurities present in the fiber are transition metal ions, such as iron, chromium, cobalt & copper. During signal propagation when photons interact with these impurity atoms, then the photons are absorbed by atoms. Hence loss occurs in light power.

- **Intrinsic absorption or internal absorption:** The fiber itself as a material has a tendency to absorb light energy however small it may be. The absorption that takes place in fiber material assuming that there are no impurities in it, is called intrinsic absorption.

(II) Scattering loss:

- When the signals travels in the fiber, the photons may be scattered due to variations in the refractive index inside the fiber. This scattering is called as Rayleigh scattering. It is also a wavelength dependent loss.

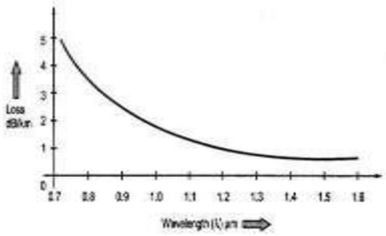
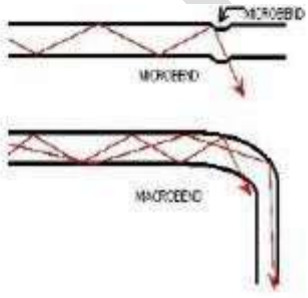


Fig. 1 Scattering loss

(III)Bending losses:

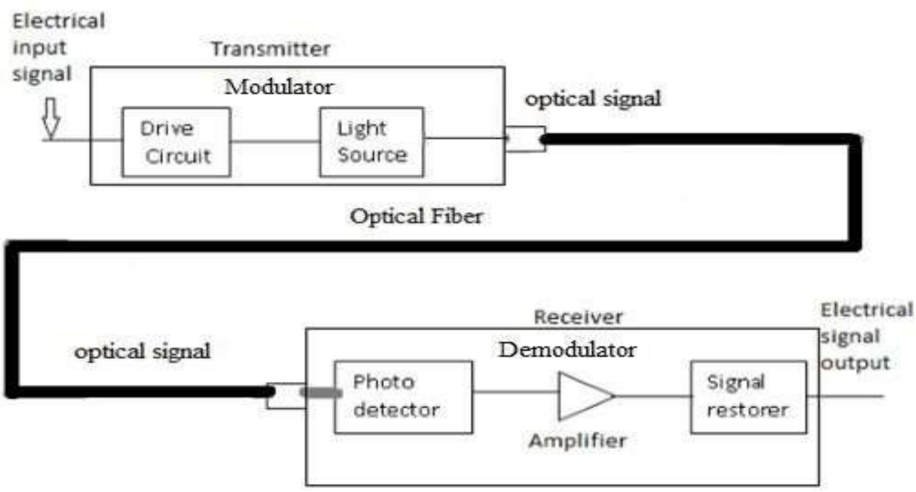
- These losses occur due to (a) Macroscopic bending (b) Microscopic bending
- **Macroscopic bending**: If the radius of core is large compared to fiber diameter causes large curvature at the bends. At these bends, the light will not satisfy the condition for total internal reflection & light escapes out from the fiber. It is called as macroscopic bending.
- **Microscopic bending**: These are caused due to non-uniform pressures created during the cabling of the fiber or during the manufacturing the fiber. It causes irregular reflections. This lead to loss of light by leakage through the fiber.



OPTICAL FIBER IN COMMUNICATION SYSTEM

The most important application of optical fibers occurs in the field of communication.

Fiber optic communication systems comprise of the following units.



Information signal source:

- The information signal to be transmitted may be voice, video or computer data (analog signals).
- In order to communicate through optical fiber, the analog signals are converted into electrical signals. (by Analog to Digital converter)
- The converted electrical signals are passed through the transmitter.

Transmitter:

- The transmitter is a modulator device used to receive electrical input signal, and then modulate it into digital pulses for propagation into an optical fiber.
- The modulator consists of a driver and a light source as shown in fig.
- The driver receives the electrical signals and then converts into the digital pulses.
- These digital pulses are converted into optical signals after passing through a light source, generally either light emitting diodes (LED's) or a semiconductor laser is used as light source.
- The optical signals are then focused into the optical fiber as shown in fig.

Optical Fiber (or) Transmission medium:

- The optical fiber is used as transmission medium between the transmitter and the receiver.
- The optical signals are then fed into an optical fiber cable where they are transmitted over long distances using the principle of total internal reflection.

Receiver:

- The receiver is a demodulator device used to receive the optical signals from the optical fiber and then convert into electrical signals.
- The demodulator consists of a photodetector, an amplifier and a signal restorer.
- The optical signals which are emerging from the optical fiber are received by photo detector.
- The photodetector converts the optical signals into electrical signals.
- The electrical signals are then amplified by the amplifier and the amplified electrical signals are converted into digital form.
- The amplified electrical signals are fed to a signal restorer where the original voice is recovered.

Advantages of Optical Fibers in communication system:

- High data transmission rates and bandwidth.
- Low losses.
- Small cable size and weight.
- Immunity to EM radiations.
- Safety due to lack of sparks.
- Data security.

SENSOR FOR STRUCTURAL HEALTH MONITORING

Optical fiber sensors (OFS) are a powerful technology for Structural Health Monitoring (SHM) of infrastructure such as bridges, tunnels, dams, buildings, aircraft, and pipelines. These sensors use the properties of light transmitted through optical fibers to detect changes in strain, temperature, vibration, and other parameters affecting structural integrity.

SHM refers to the continuous or periodic observation of a structure's condition using sensors and data analysis to:

Detect damage or degradation early

Reduce maintenance costs

Improve safety and performance

Extend the lifespan of the structure

Applications in Real-World SHM Projects

Bridges: Monitor strain, deformation, and load distribution.

Tunnels: Detect cracks and movements.

Buildings & Skyscrapers: Earthquake damage detection.

Wind Turbines & Dams: Internal stress and temperature monitoring.

Aircraft & Spacecraft: Real-time monitoring during operation.

Optical fiber sensors are ideal for SHM due to the following benefits: (the specific benefits are listed in the preceding bullet points about SHM goals).

Table: Advantages of Fiber Optic Sensors

Advantage	Explanation
Immune to EMI	Not affected by electromagnetic interference
Lightweight & Small	Easy to embed in or attach to structures
Long-range capability	Monitor kilometres of structure with one system
Multiplexing	Many sensors can be placed along a single fiber
High sensitivity	Detect minute changes in strain or temperature
Durability	Resistant to corrosion, suitable for harsh environments

Integration with Data Systems

Optical fiber sensor systems are often paired with:

Wireless sensor networks (WSNs)

Cloud platforms

AI/ML algorithms for predictive maintenance

Enables smart infrastructure with real-time alerts and analytics.

APPLICATIONS OF OPTICAL FIBERS

List of applications of optical fibers, categorized by field:

1. Telecommunications

High-Speed Internet – Fiber-optic broadband for faster and more reliable connections.

Telephone Networks – Long-distance voice transmission with minimal signal loss.

Cable TV Transmission – Delivery of digital television services.

Data Centers and Networking – Backbone of high-speed local area networks (LANs) and wide area networks (WANs).

2. Medical Applications

Endoscopy – Fiber-optic cameras for internal body imaging (e.g., gastrointestinal tract).

Laser Delivery Systems – Guiding laser beams for surgery or therapy.

Minimally Invasive Surgery – Light transmission and imaging for precision procedures.

Optical Coherence Tomography (OCT) – Non-invasive imaging of tissues, especially in ophthalmology.

3. Industrial Applications

Remote Sensing and Monitoring – Temperature, pressure, and strain measurement in harsh environments.

Welding and Machining – Fiber optics used to guide high-power lasers.

Structural Health Monitoring – Monitoring of bridges, dams, and buildings.

Oil and Gas Industry – Monitoring pipelines, wellbores, and detecting leaks.

4. Military and Aerospace

Secure Communications – Immune to electromagnetic interference (EMI) and eavesdropping.

Guided Missile Systems – Fiber guidance for signal transmission.

Aircraft and Spacecraft Systems – Lightweight, EMI-resistant data transfer.

Naval Systems – Underwater fiber-optic networks for secure communication.

5. Consumer Applications

Home Internet (FTTH) – Fiber to the Home technology for ultra-fast broadband.

Smart TVs and Streaming Devices – Fiber-optic inputs for high-definition video/audio.

Lighting (Decorative/Functional) – Fiber-optic lamps, star ceilings, holiday lighting.

6. Scientific and Research

Spectroscopy – Light delivery and collection in remote or confined locations.

Laser Delivery in Experiments – For precise light guidance in labs.

Astronomy – Fiber-fed spectrographs in telescopes for analyzing starlight.

7. Automotive and Transportation

Lighting Systems – Fiber optics used in interior and exterior lighting (e.g., dashboards, taillights).

Data Transmission – In-vehicle high-speed communication systems.

Advanced Driver-Assistance Systems (ADAS) – Using fiber optics in sensors and cameras.